ME 305  Fluid Mechanics I

Part 3

Kinematics of Fluid Flow

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Field Representation

• As a fluid moves, its properties in general change from point to point in space and from time to time.

• In field representation of a flow, fluid and flow properties are given as functions of space coordinates and time.

\[ p = p(x, y, z, t), \quad \vec{V} = \vec{V}(x, y, z, t), \quad etc. \]

• If there is no time dependency in a flow field, it is said to be steady, otherwise it is unsteady.

Movie
Steady and unsteady flows

Movie
von Karman vortex street

Movie
Wing tip vortices

Movie
Flow around a car
Different Viewpoints for Fluid and Solid Mechanics

• In solid mechanics we are usually interested in how material moves or deforms. We focus our attention on material and follow its motion/deformation.

• We locate a solid particle (or group of particles) at an initial time and study their motion in time to determine where they go.

• We are interested in particles’ trajectories and their final positions, such as golf ball’s point of hitting or maximum deflection of the beam’s center point.
Different Viewpoints for Fluid and Solid Mechanics (cont’d)

• However, in fluid mechanics we are generally interested in how things behave/change at a point, on a surface or inside a volume. We focus our attention not on material, but on space.

• For a lift force generating wing, we need to know the pressure distribution over the wing. We are not really interested in the original locations of fluid particles that cause the lift or where they go after they passed over the wing.

• To measure the amount of liquid flowing in a pipe, we need to make calculations on a certain cross section of it. We do not need to follow the fluid particles that pass through the cross section.
Lagrangian (Material) Description

Material (Lagrangian) Description: Identified fluid particles are followed in the course of time as they move in a flow field.

- NOT preferred in fluid mechanics, more suitable to solid mechanics.

- Consider a particle P flowing in the following converging duct.

<table>
<thead>
<tr>
<th>Time</th>
<th>Particle P’s speed [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$</td>
<td>5</td>
</tr>
<tr>
<td>$t_1$</td>
<td>8</td>
</tr>
<tr>
<td>$t_2$</td>
<td>10</td>
</tr>
<tr>
<td>$t_3$</td>
<td>15</td>
</tr>
<tr>
<td>$t_4$</td>
<td>20</td>
</tr>
</tbody>
</table>
Lagrangian (Material) Description (cont’d)

- In following a particle, the only independent variable is time.
- Space coordinates \((x, y, z)\) of particle P are NOT independent variables.
- When we select a particle by identifying it at its initial location at an initial time, its location at a future time, say \(t_3\), depends on which particle we are following and the value of \(t_3\).
- Properties of particle P are in general expressed as
  - position of P : \(\vec{r}_P(t)\),
  - velocity of P : \(\vec{V}_P(t)\),
  - density of P : \(\rho_P(t)\), etc.

\[ \begin{align*}
  t_0 & \quad \text{P} \\
  t_1 & \quad \text{P} \\
  t_2 & \quad \text{P} \\
  t_3 & \quad \text{P} \\
  t_4 & \quad \text{P}
\end{align*} \]

\(\text{Lagrangian description} \)
Eulerian (Spatial) Description

Spatial (Eulerian) Description: Attention is focused at fixed points (or area or volume) in the flow field and the variation of properties at these points (or area or volume) are determined as fluid particles pass through these points.

• This is the preferred viewpoint for fluid mechanics.
• Consider the same duct flow, but now concentrating at two points, A and B.
Eulerian (Spatial) Description (cont’d)

• Now both time and space coordinates are independent variables.

• Location of point A (or B) does NOT depend on the flow field or time.

• Fluid and flow properties at a point (e.g. point A) are expressed as

  velocity : \( \mathbf{V}_A(x_A, y_A, z_A, t) \),
  density : \( \rho_A(x_A, y_A, z_A, t) \),
  pressure : \( p_A(x_A, y_A, z_A, t) \), etc.

• The duct flow described in the previous slides is said to be **steady** if the flow properties (such as velocity) do not change with time.

• For steady flows time is NOT a variable in the Eulerian description.

• But time is always an independent variable in the Lagrangian description, even for steady flows. Without time, a fluid particle simply cannot move.
Lagrangian vs. Eulerian Description

Exercise: Are the following descriptions Lagrangian or Eulerian?

• A doctor using X-ray opaque dye to trace blood flow in arteries.
• A civil engineer studying the traffic load of a highway by focusing at a certain section of the road and counting the number of cars passing in front of him during a certain period of time.
• A student performing wind tunnel experiment and measuring the velocity at different points of a flow field by manually moving a velocity measuring probe.
• Fluid dynamic measurements performed in the lab are suited to the Eulerian description. A velocity or pressure probe inserted in a flow field do NOT move with the flow, but provide data at the locations we point it to.

Exercise: Visit Storm Chaser’s web site to see the use of a Lagrangian type probe for gathering data inside a twister. Also you can watch the movie Twister to see such probes in action.
Use of Eulerian Description for Solid Mechanics

• Eulerian description is also preferred in studying very high deformation solid mechanics problems in which solids show fluid-like behavior.

Exercise: Do a research on the working principle of “shaped charge” used for armor penetration. Watch the movie
http://www.youtube.com/watch?v=LudNqf56AFo

Exercise: Watch the following movies in which solids undergo very excessive (fluid-like) deformation.

Aluminum extrusion: http://www.youtube.com/watch?v=9mQ2ic-kDlk

Deep drawing: http://www.youtube.com/watch?v=PBB3utteDq0

Crash test: http://www.youtube.com/watch?v=CcXhjH0hex0
Use of Lagrangian Description for Fluid Mechanics

• A doctor using X-ray opaque dye to trace blood flow in arteries performs a Lagrangian study.

• In some Computational Fluid Dynamics (CFD) studies motion of fluid particles are modeled in a Lagrangian way.

Exercise: Watch this particle simulation of flow around a car

http://www.youtube.com/watch?v=RuZQpWo9Qhs

Exercise: RealFlow is a particle based fluid simulation software used in film-making and television industry. Visit http://www.realflow.com/rf_casestudies_portal.php to see its capabilities.

Movie
SPH Simulation of a tanker in wave

Movie
RealFlow Demo Reel
Differential vs. Integral Formulation

- **Differential formulation** provides a very detailed solution of a flow field.
- When coupled with Eulerian point of view, it provides information at all points in the problem region at all times of interest.
- It requires the solution of differential equations for conservation laws (mass, momentum and energy).
- Analytical solution of conservation equations are available only for a few very simple problems. **Computational Fluid Dynamics (CFD)** provides an alternative.
Differential vs. Integral Formulation (cont’d)

- **Integral formulation** when coupled with Eulerian viewpoint focuses at a fixed region of space (control volume).
- It provides less information compared to differential approach.
- But it has much simpler mathematics. No DE solution is required.
- It is used to determine **gross flow effects (not details)**, such as the lift force generated by a wing or the shaft work required to run a pump.
Lagrangian - Eulerian Relation

- Consider a property $N$ in a flow field.
- At time $t$ fluid particle $P$ passes through a point $A$ in space.

We want to determine a relation in the following form.

\[
\text{Rate of change of property } N \text{ of particle } P \text{ at time } t \text{ from a Lagrangian point of view} = \text{Rate of change of property } N \text{ at point } A \text{ from an Eulerian point of view}
\]
Lagrangian - Eulerian Relation (cont’d)

- Property $N$ in Eulerian description is given as $N(x, y, z, t)$
- Total change of this property is

$$dN = \frac{\partial N}{\partial t} dt + \frac{\partial N}{\partial x} dx + \frac{\partial N}{\partial y} dy + \frac{\partial N}{\partial z} dz$$

- Change of $N$ in both time and space
- Change of $N$ in time
- Change of $N$ in space

- Divide both sides by $dt$

$$\frac{dN}{dt} = \frac{\partial N}{\partial t} dt + \frac{\partial N}{\partial x} dx + \frac{\partial N}{\partial y} dy + \frac{\partial N}{\partial z} dz$$

$$\frac{dN}{dt} = \frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} + w \frac{\partial N}{\partial z}$$

$$\left( \vec{V} \cdot \nabla \right) N$$
Lagrangian - Eulerian Relation (cont’d)

\[
\frac{dN}{dt} = \frac{\partial N}{\partial t} + (\vec{V} \cdot \nabla)N
\]

- **Material derivative**: Rate of change of property \(N\) in the material description (following a particle)
- **Partial derivative**: Rate of change of property \(N\) with time only. For a steady flow this term is zero for any property.
- **Convective derivative**: Change of property \(N\) with respect to space only, i.e. at a fixed time. If there is no flow this term is zero, i.e. properties of a stationary fluid particle may change with time only.
Lagrangian - Eulerian Relation (cont’d)

\[
\frac{dN}{dt} = \frac{\partial N}{\partial t} + (\bar{V} \cdot \nabla)N
\]

- Steady state operation of a water heater.
- Fluid heats up in the heater.
- \(\frac{\partial T}{\partial t}\) of any fluid particle is zero, but \(d\frac{T}{dt}\) is not zero.
- Convective derivative of \(T\) is not zero.

- Steady state uniform flow in a converging-diverging nozzle.
- Fluid particles first accelerate and then decelerate.
- \(\frac{\partial u}{\partial t}\) of a fluid particle is zero, but \(du/dt\) is not zero.
- Convective derivative of \(u\) is not zero.
Acceleration of a Fluid Particle

• Selecting $N = \vec{V}$ in equation $\frac{dN}{dt} = \frac{\partial N}{\partial t} + (\vec{V} \cdot \nabla)N$ acceleration of a fluid particle can be obtained as

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}$$

Local acceleration

Convective acceleration

• Components of the acceleration vector in Cartesian coordinate system are

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + (\vec{V} \cdot \nabla)u = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial t} + (\vec{V} \cdot \nabla)v = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial t} + (\vec{V} \cdot \nabla)w = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$
Exercise: Using the following \( \nabla \) operator in the cylindrical coordinate system

\[
\nabla = \frac{\partial}{\partial r} \vec{i}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \vec{i}_\theta + \frac{\partial}{\partial z} \vec{i}_z
\]

and the fact that in cylindrical coordinate system unit vectors have the following non-zero derivatives

\[
\frac{\partial \vec{i}_r}{\partial \theta} = \vec{i}_\theta \quad \text{and} \quad \frac{\partial \vec{i}_\theta}{\partial \theta} = -\vec{i}_r
\]

derive the following acceleration components

\[
a_r = \frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_r}{\partial \theta} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r}
\]

\[
a_\theta = \frac{\partial V_\theta}{\partial t} + V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_\theta}{\partial \theta} + V_z \frac{\partial V_\theta}{\partial z} + \frac{V_r V_\theta}{r}
\]

\[
a_z = \frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z}
\]
Exercise: Consider one-dimensional, steady, incompressible flow through a converging channel.

The velocity field is given by \( u = U_o(1 + x/L) \), \( v = 0 \) and \( w = 0 \).

a) Determine the acceleration field, \( a(x) \), by using the Eulerian method.

b) Using the Lagrangian method, determine the equations for the position and acceleration of the fluid particle as a function of time, which is located at \( x = 0 \) at time \( t = 0 \).

c) Show that both expressions for the acceleration give identical results, as the fluid particle passes through the point at \( x = L \).
Flow Classification as 1D, 2D and 3D

- Depends on the number of space coordinates \((x, y, z)\) required to specify the velocity field.

- **3D Flow**: \(\vec{V} = \vec{V}(x, y, z)\)

- **2D Flow**: \(\vec{V} = \vec{V}(x, y)\) or \(\vec{V} = \vec{V}(x, z)\) or \(\vec{V} = \vec{V}(y, z)\)

- **1D Flow**: \(\vec{V} = \vec{V}(x)\) or \(\vec{V} = \vec{V}(y)\) or \(\vec{V} = \vec{V}(z)\)

- For some problems use of \((r, \theta, z)\) is more suitable than \((x, y, z)\).

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**Diagram:**

- **Fully developed flow in a constant diameter pipe:**
  - **1D flow**: \(\vec{V}(r)\)

- **Decelerating flow in a pipe of increasing diameter:**
  - **2D flow**: \(\vec{V}(r, z)\)

- **Axisymmetric flow. Nothing depends on \(\theta\)**
Flow Classification as Steady, Unsteady

• **Steady flow**: Local derivatives ($\partial / \partial t$) are zero in a flow field. Properties at a fixed point do not change in time.
  - See slide 3-17 for two examples.
  - A centrifugal pump working constantly at the same speed between the same input and output conditions is said to be working steadily.

• **Unsteady flow**: Local derivatives of properties are nonzero. Properties at a fixed point change in time.
  - If the inlet water temperature of the heater shown in slide 3-17 changes with time, it will be a unsteady flow.
  - Pulsatile blood flow in our veins is unsteady. But it is a special kind of unsteady flow, it is *time periodic*. It repeats itself after a certain period.
  - von Karman vortex street of slide 3-2 is also unsteady and time periodic.
  - A gusty wind blowing over a house is unsteady.
Flow Classification as Steady, Unsteady (cont’d)

• Sometimes an unsteady flow can be studied as steady by a proper choice of reference frame.

• Consider the following wing moving at a constant speed in still air.

• For an observer fixed at the ground this flow is unsteady.

• At an upstream point A, initially air speed is zero. But as the wing approaches point A, it will push the air there. Observer fixed at the ground sees different things at point A (and also at other points) at different times.

• The same flow becomes steady with respect to an observer moving with the wing.

• This observer will always see the same air motion around her. Air will always approach with the same speed. Nothing will change in time, except her wrinkles if she watches the air flow for too long.
Flow Classification as Laminar, Turbulence

- **Laminar flow** is a well-ordered state of flow in which adjacent fluid layers move smoothly with respect to each other.
  - Laminar flow is usually associated with low speeds and high viscosities.
- **Turbulent flow** has random, unsteady fluctuations. It is inherently 3D. There is intense mixing and rotationality.
  - Turbulent flow is usually associated with high speeds.
  - Turbulent flows are, by far, the most common.
  - Although a turbulent flow always have unsteadiness in it, it may be steady in the mean (in a time averaged sense).

*Movie*

Laminar flow over cylinders and airfoils

Laminar to turbulent transition in a pipe

Wake behind a cylinder
Pathlines, Streaklines and Streamlines

• These are three different flow visualization techniques.
• **Pathline** is a line traced out by a fluid particle as it flows in a flow field.
  • Pathline is a Lagrangian concept.
  • In laboratory it can be generated by marking (dying) a small fluid element and taking time exposure photograph of its motion.
• **Streakline** consists of all particles in a flow that have previously passed through a common point.
  • In laboratory it can be generated by continuously injecting dye at a point and observing the collection of dyed particles as they move in the flow.
• **Streamline** is a line that is everywhere tangent to the velocity field.
  • It is a mathematical tool, rather than a laboratory one.

• For a **steady flow** all these three are the same.
• For an **unsteady flow** they are all different.

*Movie*
Pathline, streakline and streamline comparison for unsteady oscillating plate flow
Obtaining Streamlines Mathematically

• Consider a streamline in a 2D flow field.
• At any point velocity vector will be tangent to it.
• Slope of the line at any point \( \frac{dy}{dx} \) should be equal to the velocity component ratio \( \frac{v}{u} \)

\[
\frac{dy}{dx} = \frac{v}{u} \]

which can also be written as

\[
\frac{dx}{u} = \frac{dy}{v} \]

• This can be generalized to a 3D flow as

\[
\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \]

• If the velocity field is known as a function of \( x, y \) and \( z \) (and \( t \) if the flow is unsteady), the above equation can be integrated to give the equation of streamlines.

**Exercise:** For the velocity field given by \( \vec{V} = 2x \hat{i} - 2y \hat{j} \), determine the equation of the streamline that passes through point \( P(2,2,0) \).
Closed System vs. Control Volume

- A closed system (or just system) is a fixed, identifiable quantity of mass.
- It can change its position and shape, but it always contains the same fluid particles.
- It is separated from the surroundings by system boundaries, which is closed to mass transfer. Fluid particles can not pass through it.
- It is related to the Lagrangian point of view.
- It has the advantage that basic laws (conservation of mass, momentum, energy) can be written for it in a very natural and simple way.

Initially the closed system for the perfume is inside the spray can

After using, the closed system is partly inside and partly outside. It follows perfume’s motion.
Closed System vs. Control Volume (cont’d)

- A control volume (CV) is a fixed region of a flow field.
- It can NOT change its position or shape, but it contains different fluid particles at different times.
- It is separated from the surroundings by control surface (CS), which is open to mass transfer. Fluid particles can pass through it.
- It is related to the Eulerian point of view.
- Reynolds Transport Theorem (RTT) is used to convert basic laws into a form suitable to CV use.

![Initial shape of the CV.](image)

![After using, CV does not change shape. It does not follow perfume’s motion. It has an exit, through which fluid leaves.](image)
Basic Laws Written for a System

Conservation of Mass: Mass of a closed system does not change, i.e. time rate of change of a closed system’s mass is zero.

\[
\frac{dm_{sys}}{dt} = 0 \quad \text{where} \quad m_{sys} = \int_{\forall_{sys}} \rho \, d\forall
\]

Conservation of Linear Momentum (Newton’s 2\textsuperscript{nd} Law): Sum of all external forces acting on a system is equal to the time rate of change of linear momentum.

\[
\sum \vec{F} = \frac{d\vec{P}_{sys}}{dt} \quad \text{where} \quad \vec{P}_{sys} = \int_{\forall_{sys}} \rho \, \vec{V} \, d\forall
\]

Conservation of Angular Momentum: Sum of all external torques acting on a system is equal to the time rate of change of angular momentum.

\[
\sum \vec{\tau} = \frac{d\vec{H}_{sys}}{dt} \quad \text{where} \quad \vec{H}_{sys} = \int_{\forall_{sys}} \rho \left( \vec{r} \times \vec{V} \right) \, d\forall
\]
Basic Laws Written for a System (cont’d)

Conservation of Energy (1st Law of Thermodynamics): Energy of a closed system changes by heat and work interaction with its surroundings as follows

\[ \dot{Q} + \dot{W} = \frac{dE_{sys}}{dt} \]

where

\[ E_{sys} = \int_{\forall_{sys}} \rho \ e \ d\forall \]

\[ e = u + \frac{V^2}{2} + gz \]

\( \dot{Q} \): rate of heat transfer (heat coming into the system is positive)

\( \dot{W} \): rate of work done (work done on the system is positive)
Reynolds Transport Theorem (RTT)

- All basic laws are naturally written for a closed system.
- But we want to use CVs to study fluid mechanics problems.
- RTT is a general relation between the rate of change of a fluid property in a closed system and the corresponding control volume.

\[
\frac{dN_{\text{sys}}}{dt} = \frac{d}{dt} \int_{\text{sys}} \rho \eta \, d\mathcal{V} = \frac{\partial}{\partial t} \int_{\text{CV}} \rho \eta \, d\mathcal{V} + \int_{\text{CS}} \rho \eta \, (\mathbf{V} \cdot \mathbf{n}) \, dA
\]

where \(N\) is any extensive property such as \(m\), \(\bar{P}\), \(\bar{H}\) or \(E\).

\(\eta\) is the corresponding intensive property such as \(1\), \(\bar{V}\), \((\mathbf{r} \times \mathbf{V})\) or \(e\).

\(\mathbf{V}\) is the velocity of the fluid at the control surface (CS) and \(\mathbf{n}\) is the unit outward normal of the CS.

**Exercise:** Study the derivation of RTT from the distributed handout.
Reynolds Transport Theorem (cont’d)

\[
\frac{dN_{sys}}{dt} = \frac{d}{dt} \int_{sys} \rho \eta \, d\mathcal{V} = \frac{\partial}{\partial t} \int_{CV} \rho \eta \, d\mathcal{V} + \int_{CS} \rho \eta (\mathbf{V} \cdot \mathbf{n}) \, dA
\]

- We do not evaluate the left hand side directly, instead use the equivalents such as
  - Zero : for mass conservation
  - \( \sum \mathbf{F} \) : for linear momentum conservation
  - \( \sum \mathbf{T} \) : for angular momentum conservation
  - \( \dot{Q} + \dot{W} \) : for energy conservation
Let’s apply RTT to the spray can example.

For mass conservation, $N = m$ and $\eta = 1$

$$\frac{dm_{sys}}{dt} = \frac{\partial}{\partial t} \int_{CV} \rho \, d\forall + \int_{CS} \rho \, (\vec{V} \cdot \vec{n}) \, dA$$

Zero = Time rate of change of perfume's mass inside the spray can. A negative value.

Amount of mass that leaves the can in unit time. A positive value (Nonzero only at the little opening that the gas can escape from)