LECTURE NOTES FOR WEB LEARNING

Stability of Earth Slopes

Unit-6
(08CV64)

BY
Dr. S V Dinesh
Professor
Civil Engineering Department
Siddaganga Institute of Technology
Tumkur
1. Introduction:

An exposed ground surface that stands at an angle ($\beta$) with the horizontal is called slope. Slopes are required in the construction of highway and railway embankments, earth dams, levees and canals. These are constructed by sloping the lateral faces of the soil because slopes are generally less expensive than constructing walls. Slopes can be natural or man made. When the ground surface is not horizontal a component of gravity will try to move the sloping soil mass downwards. Failure of natural slopes (landslides) and man-made slopes has resulted in much death and destruction. Some failures are sudden and catastrophic; others are widespread; some are localized. Civil Engineers are expected to check the safety of natural and slopes of excavation. Slope stability analysis consists of determining and comparing the shear stress developed along the potential rupture surface with the shear strength of the soil. Attention has to be paid to geology, surface drainage, groundwater, and the shear strength of soils in assessing slope stability.

In this chapter, we will discuss simple methods of slope stability analysis from which one will be able to:

- Understand the forces and activities that provoke slope failures.
- Understand the effects of seepage on the stability of slopes.
- Estimate the stability of slopes with simple geometry for different types of soils.

Man made slopes are used in

- Highways
- Railways
- Earthdams
- River Training works

### 1.2 Slope Failure Triggering Mechanisms

- Intense Rain-Fall
- Water-Level Change
- Seepage Water Flow
- Volcanic Eruption
- Earthquake Shaking
- Human activity

### 1.3 Causes of Slope failure

1. **Erosion**: The wind and flowing water causes erosion of top surface of slope and makes the slope steep and thereby increase the tangential component of driving force.

2. **Steady Seepage**: Seepage forces in the sloping direction add to gravity forces and make the slope susceptible to instability. The pore water pressure decrease the shear strength. This condition is critical for the downstream slope.

3. **Sudden Drawdown**: in this case there is reversal in the direction flow and results in instability of side slope. Due to sudden drawdown the shear stresses are more due to saturated unit weight while the shearing resistance decreases due to pore water pressure that does not dissipate quickly.

4. **Rainfall**: Long periods of rainfall saturate, soften, and erode soils. Water enters into existing cracks and may weaken underlying soil layers, leading to failure, for example, mud slides.

5. **Earthquakes**: They induce dynamic shear forces. In addition there is sudden buildup of pore water pressure that reduces available shear strength.

6. **External Loading**: Additional loads placed on top of the slope increases the gravitational forces that may cause the slope to fail.
7. **Construction activities at the toe of the slope:** Excavation at the bottom of the sloping surface will make the slopes steep and thereby increase the gravitational forces which may result in slope failure.

### 1.4 Types of failure

Broadly slope failures are classified into 3 types as:

1. **Face (Slope) failure**
2. **Toe failure**
3. **Base failure**

   1. **Face (Slope) Failure:** This type of failure occurs when the slope angle ($\beta$) is large and when the soil at the toe portion is strong.

   ![Face Failure Diagram](image)

   2. **Toe Failure:** In this case, the failure surface passes through the toe. This occurs when the slope is steep and homogeneous.

   ![Toe Failure Diagram](image)

   3. **Base Failure:** In this case, the failure surface passes below the toe. This generally occurs when the soil below the toe is relatively weak and soft.

   ![Base Failure Diagram](image)

### 1.5 Definition of Key Terms
**Slip or failure zone:** It is a thin zone of soil that reaches the critical state or residual state and results in movement of the upper soil mass.

**Slip plane or failure plane or slip surface or failure surface:** It is the surface of sliding.

**Sliding mass:** It is the mass of soil within the slip plane and the ground surface.

**Slope angle (β):** It is the angle of inclination of a slope to the horizontal. The slope angle is sometimes referred to as a ratio, for example, 2:1 (horizontal: vertical).
1.6 Stability Analysis consists of

- Determination of the potential failure surface.
- Forces that tend to cause slip.
- Forces that tend to restore (stabilize)
- Determination of the available margin of safety.

1.7 Types of Slopes

1. Infinite Slopes
2. Finite Slopes

1.7.1 Infinite slopes: They have dimensions that extend over great distances and the soil mass is inclined to the horizontal.

1.7.2 Finite slopes: A finite slope is one with a base and top surface, the height being limited. The inclined faces of earth dams, embankments and excavation and the like are all finite slopes.

1.8 Factor of safety

Factor of safety of a slope is defined as the ratio of average shear strength ($\tau_f$) of a soil to the average shear stress ($\tau_d$) developed along the potential failure surface.

$$FS = \frac{\tau_f}{\tau_d}$$

FS = Factor of safety

$\tau_f$ = average shear strength of the soil


\( \tau_d = \) average shear stress developed along the potential surface.

**Shear Strength:**

Shear strength of a soil is given by

\[
\tau_f = c + \sigma \tan \phi
\]

Where, \( c = \) cohesion

\( \Phi = \) angle of internal friction

\( \sigma = \) Normal stress on the potential failure surface

Similarly, the mobilized shear strength is given by

\[
\tau_d = c_d + \sigma \tan \phi_d
\]

\( c_d \) and \( \phi_d \) are the cohesion and angle of internal friction that develop along the potential failure surface.

\[
FS = \frac{c + \sigma \tan \phi}{c_d + \sigma \tan \phi_d}
\]

FS w.r.t cohesion is

\[
F_c = \frac{c}{c_d}
\]

\[
F_\phi = \frac{\tan \phi}{\tan \phi_d}
\]

When \( F_c = F_\phi \) it gives Factor of safety w.r.t strength

\[
\frac{c}{c_d} = \frac{\tan \phi}{\tan \phi_d}
\]

Then \( F_s = F_c = F_\phi \)

When \( FS = 1 \), then the slope is said to be in a state of failure.

**1.9 Infinite Slopes:**

Infinite slopes have dimensions that extended over great distances and the soil mass is inclined to the horizontal. If different strata are present strata boundaries
are assumed to be parallel to the surface. Failure is assumed to occur along a plane parallel to the surface.

Fig 2: Infinite Slope in layered soils

3 cases of stability analysis of infinite slopes are considered
Case (i) Cohesionless soil
Case (ii) Cohesive soil
Case (iii) Cohesive-frictional soil.

1.9.1 Infinite slopes in cohesionless soils

Fig 3: Infinite slope in cohesionless soil
Consider an infinite slope in a cohesionless soil inclined at an angle $\beta$ to the horizontal as shown. Consider an element ‘abcd’ of the soil mass.

Let the weight of the element be $W$.

The component of $W$ parallel to slope = $T = W \sin \beta$

The component of $W$ perpendicular to slope = $N = W \cos \beta$

The force that causes slope to slide = $T = W \sin \beta$

The force that restrains the sliding of the slope = $\sigma \tan \phi$

\[ = N \tan \phi = W \cos \beta \tan \phi \]

The factor of safety against sliding failure is

\[ FS = \frac{\text{Restraining force}}{\text{Sliding force}} = \frac{W \cos \beta \tan \phi}{W \sin \beta} \]

\[ FS = \frac{\tan \phi}{\tan \beta} \]

under the limiting equilibrium $FS = 1$

\[ \tan \beta = \tan \phi \]

\[ \beta = \phi \]

“The maximum inclination of an infinite slope in a cohesionless soil for stability is equal to the angle of internal friction of the soil”.

The limiting angle of inclination for stability of an infinite slope in cohesionless soil is as shown below.

- $\beta < \phi$ - Slope is stable
- $\beta = \phi$ - Limiting condition
- $\beta > \phi$ - Unstable
Fig 4: Normal stress Vs Shear stress indicating the limiting condition of a slope

1.9.2 Effect of seepage when seepage force is parallel to the slope

![Diagram of infinite slope in cohesionless soil under steady seepage]

Consider an infinite slope AB with steady seepage parallel to the sloping surface. Due to this seepage force \( J_s \) acts in the direction of seepage

Seepage force \( J_s = \gamma_w b_j Z_j \)

\( i = \sin \beta \) (Since seepage is parallel to the slope)

\[ N' = W' \cos \beta = \gamma' b_j Z_j \cos \beta \]

\[ T = W' \sin \beta + J_s = \gamma' b_j Z_j \sin \beta + \gamma_w b_j Z_j \sin \beta \]

\[ = (\gamma' + \gamma_w) b_j Z_j \sin \beta = \gamma_{sat} b_j Z_j \sin \beta \]

\[ FS = \frac{\text{Restraining force}}{\text{Sliding force}} = \frac{N' \tan \phi'}{T} \]

\[ FS = \frac{\gamma' b_j Z_j \cos \beta \tan \phi'}{\gamma_{sat} b_j Z_j \sin \beta} = \frac{\gamma' \tan \phi'}{\gamma_{sat} \tan \beta} \]

At limit equilibrium \( F_s = 1 \)

\[ \tan \beta = \frac{\gamma'}{\gamma_{sat} \tan \phi'} \]
\[
\frac{\gamma'}{\gamma_{sat}} = \frac{1}{2} \text{ (approximately)}
\]

\[\therefore \tan \beta = \frac{1}{2} \tan \phi'\]

“The seepage parallel to the slope reduces the limiting slope angle in coarse grained soil by one-half of the friction angle”.

1.9.3 Infinite slope in pure cohesive soil

Fig 7: Free body diagram of slice in infinite slope of pure cohesive soil

The normal stress on the failure surface is \(\sigma_n = \frac{N}{l \times 1}\)

\[\cos \beta = \frac{b}{l} ; \quad l = \frac{b}{\cos \beta}\]

\[\sigma_n = \frac{W \cos \beta}{b} = \frac{W}{b} \cos^2 \beta\]

\[
\frac{\cos \beta}{b}
\]
\[ W = \gamma Z b \]
\[ \sigma_n = \frac{\gamma Z b}{b} \cos^2 \beta = \gamma Z \cos^2 \beta \]
\[ \sigma_n = W \cos^2 \beta \]

The shear stress \( \tau_d \) can be expressed as:
\[ \tau_d = \frac{T}{1 \times 1} = \frac{W \sin \beta}{1 \times 1} = \frac{W \sin \beta}{b \cos \beta} \]
\[ \tau_d = \frac{W}{b} \sin \beta \cos \beta \]
\[ \tau_d = \frac{\gamma Z b}{b} \sin \beta \cos \beta \]
\[ \tau_d = \gamma Z \sin \beta \cos \beta \]

The Mohr Coulomb shear stress is:
\[ \tau_f = c + \sigma \tan \phi = c \quad \text{(cohesive soil)} \]
\[ FS = \frac{\tau_f}{\tau_d} = \frac{c}{\gamma Z \sin \beta \cos \beta} \]
\[ FS = \frac{c}{\gamma Z \sin \beta \cos \beta} \]

At critical conditions \( FS = 1 \)
\[ Z_c = \frac{c}{\gamma \sin \beta \cos \beta} \]

For a given slope angle \( \phi \), \( Z_c \) is directly proportional to the cohesion and inversely proportional to unit weight.

\( \frac{c}{\gamma Z_c} \) is a dimensionless quantity called the stability number denoted by \( S_n \).
\[ S_n = \frac{c}{\gamma Z_c} \]

we know

\[ FS = \frac{c}{\gamma Z \sin \beta \cos \beta} \]

\[ \sin \beta \cos \beta = \frac{c}{\gamma Z_c} \]

\[ FS = \frac{c}{\gamma Z} \cdot \frac{c}{\gamma Z_c} = \frac{Z_c}{Z} \]

\[ FS = \frac{Z_c}{Z} \]

1.9.4 Infinite slope in cohesive frictional soil

Fig 8: Infinite slope in c-φ soil
Consider an infinite slope in c-φ soil as shown with slope angle $\beta$

The strength envelope for the c-φ soil is $\tau = c + \sigma_{nf} \tan \phi$. If the slope angle $\beta$ is less than $\phi$, slope will be stable for any depth.

When the slope angle $\beta > \phi$, the slope will be stable upto a depth $Z=Z_c$ corresponding to point P. The point P corresponds to the depth at which the shear stress mobilized will be equal to the available shear strength.

For all depths less than that represented by point P shearing stress will be less than the shear strength and the slope will be stable

\[ \tau = c + \sigma \tan \phi \]

\[ \tau_f = c + \sigma \tan \phi \]

At P:

\[ \sigma_{nf} = \gamma Z_c \cos^2 \beta \]

\[ \tau_f = c + \gamma Z_c \cos^2 \beta \tan \phi \quad \text{----- 1} \]

the developed shear stress is

\[ \tau_d = \gamma Z_c \sin \beta \cos \beta \quad \text{----- 2} \]

Equating 1 and 2

\[ \gamma Z_c \sin \beta \cos \beta = c + \gamma Z_c \cos^2 \beta \tan \phi \]
\[
\gamma Z_c \left( \sin \beta \cos \beta - \cos^2 \beta \tan \phi \right) = c
\]

\[
\gamma Z_c \cos^2 \beta \left( \frac{\sin \beta}{\cos \beta} - \tan \phi \right) = c
\]

\[
\gamma Z_c \cos^2 \beta (\tan \beta - \tan \phi) = c
\]

\[
Z_c = \frac{c}{\gamma \cos^2 \beta (\tan \beta - \tan \phi)}
\]

Therefore the critical depth \(Z_c\) is proportional to cohesion for a given value of slope angle (\(\beta\)) and friction angle (\(\phi\)).

Therefore \(\frac{c}{\gamma Z_c} = \cos^2 \beta (\tan \beta - \tan \phi)\)

The term \(\frac{c}{\gamma Z_c} = S_n\), Stability number (\(S_n\)).

For all depth \(Z < Z_c\)

\[
FS = \frac{\text{Shearing strength}}{\text{shearing stress}} = \frac{c + \gamma Z \cos^2 \beta \tan \phi}{\gamma Z \sin \beta \cos \beta}
\]

### 1.10 Finite Slopes

A finite slope is one with a base and top surface, the height being limited. The inclined faces of earth dams, embankments, excavation and the like are all finite slopes.

Investigation of the stability of finite slopes involves the following steps:

a) assuming a possible slip surface,

b) studying the equilibrium of the forces acting on this surface, and

c) Repeating the process until the worst slip surface, that is, the one with minimum margin of safety is found.

**Methods:**

I. Total stress analysis for purely cohesive soil.
II. Total stress analysis for cohesive – frictional (c-\( \phi \)) soil – (Swedish method of slices or Method of slices)

III. Effective stress analysis for conditions of steady seepage, rapid drawdown and immediately after construction.

IV. Friction circle method

V. Taylor’s method.

1.10.1 Total stress analysis for pure cohesive soil

The analysis is based on total stresses, it is also called \( \phi = 0 \) analysis. It gives the stability of an embankment immediately after construction. It is assumed that the soil has no time to drain and the shear strength parameters used are obtained from undrained conditions with respect to total stresses. These may be obtained from either unconfined compression test or an undrained triaxial test without pore pressure measurements.

Let AB be a trial slip surface in the form of a circular arc of radius ‘r’ with respect to center of rotation ‘O’ as shown in Fig-10.
Let ‘W’ be the weight of the soil within the slip surface

Let ‘G’ be the position of its centre of gravity.

The exact position of G is not required and it is only necessary to ascertain the position of the line of action of W, this may be obtained by dividing the failure plane into a set of vertical slices and taking moments of area of these slices about any convenient vertical axis.

The shearing strength of the soil is c, since $\phi = 0^\circ$.

The restoring moment (along the slip surface) = $c lr$

$$= cr\theta r = cr^2 \theta$$

The driving moment = $W.e$

Factor of safety, $FS = \frac{\text{Restoring moment}}{\text{Driving moment}} = \frac{cr^2 \theta}{W.e}$

1.11 Effect of Tension cracks on Stability

In case of cohesive soil when the slope is on the verge slippage there develops a tension crack at the top of the slope as shown in Fig 11. the depth of tension crack is

Fig 11: Infinite slope with tension crack on top
\[ hc = \frac{2c}{\gamma} \]

Where, \( c = \) cohesion

\( \gamma = \) unit weight

There is no shear resistance along the crack. The failure arc reduces from Arc AB to Arc AB' and the angle \( \theta \) reduces to \( \theta' \).

For computation of FS we have to

1. Use \( \theta' \) instead of \( \theta \) in the restoring moment component.
2. Consider the full weight ‘\( W \)’ of the soil within the sliding surface AB to compensate for filling of water in the crack in the driving moment component

\[ FS = \frac{c.r.\theta'}{W.e} \]

"Tension crack reduces F.S due to decrease in restoring moment"

The effect of tension cracks are

1. It modifies the slip surface and reduces the length of the slip surface.
2. It is usually filled with water and produces hydrostatic pressure along the depth.
3. It acts as channel for water to flow into underlying soil layers, inducing seepage forces.
4. It reduces the factor of safety.

1.12 The Swedish method of slices for a cohesive–frictional (c-\( \phi \)) soil
For c-φ soils the undrained strength envelope shows both c and φ values. The total stress analysis can be adopted.

The procedure is follows:

1. Draw the slope to scale
2. A trial slip circle such as AB with radius ‘r’ is drawn from the center of rotation O.
3. Divide the soil mass above the slip surface into convenient number of slices (more than 5 is preferred)
4. Determine the area of each slice A1, A2, ------, An
   \[ A = \text{width of the slice} \times \text{mid height} \]
   \[ = b \times Z \]
5. Determine the total weight W including external load if any as
   \[ W = \gamma b Z = \gamma A \]

Where, \( \gamma \) = unit weight
b = width of slice
Z = height of slice.

The forces on a typical slice are given in Fig 12.
The reactions $R_1$ and $R_2$ on the sides of the slice are assumed equal and therefore do not have any effect on stability.

6. The weight $W$ of the slice is set off at the base of the slice. The directions of its normal component ‘$N$’ and the tangential component ‘$T$’ are drawn to complete the vector triangle.

$$N = W \cos \delta, \quad T = W \sin \delta$$

7. The values of $N$ and $T$ are scaled off for each of the slices

8. The values of ‘$N$’ and ‘$T$’ are tabulated and summed up as shown in the following table.

9. The factor of safety is calculated as follows

$$\text{Sliding moment} = r \sum T \quad (\text{reckoned positive if clockwise})$$

$$\text{Restoring moment} = r (c r \theta + \sum N \tan \phi) \quad (\text{reckoned positive if counterclockwise})$$

$$\text{Factor of safety}, FS = \frac{(c r \theta + \sum N \tan \phi)}{\sum T}$$

Note: The tangential components of a few slices at the base may cause restoring moments.

**Table 1: Normal and tangential components of various slices in the slope**

<table>
<thead>
<tr>
<th>Slice No.</th>
<th>Area $m^2$</th>
<th>Weight $W$ (kN)</th>
<th>Normal component $N$ (kN)</th>
<th>Tangential components $T$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>$N = W \cos \delta$</td>
<td>$T = W \sin \delta$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Sum, $\Sigma N = \ldots$ kN | Sum, $\Sigma T = \ldots$ kN |

10. Repeat step 2 to 9 by considering various trial slip circles and calculate $FS$ for each of these slip circles. The slip circle with a minimum $FS$ is called critical slip circle.
1.13 Critical Slip Circle by Fellenius Direction angles

In case of slopes in homogeneous cohesive soil deposits, the centre of a critical circle can be directly located by using Fellenius direction angles. Fellenius (1936) has given direction angles $\alpha$ and $\beta$ for various slopes as shown below.

![Fig 13: Fellenius direction angles in finite slope](image)

Table 2: Fellenius direction angles for locating critical slip circle

<table>
<thead>
<tr>
<th>Slope</th>
<th>Angle $\alpha$</th>
<th>Angle $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1</td>
<td>28°</td>
<td>37°</td>
</tr>
<tr>
<td>1:1.5</td>
<td>26°</td>
<td>35°</td>
</tr>
<tr>
<td>1:2</td>
<td>25°</td>
<td>35°</td>
</tr>
<tr>
<td>1:3</td>
<td>25°</td>
<td>35°</td>
</tr>
<tr>
<td>1:5</td>
<td>25°</td>
<td>37°</td>
</tr>
</tbody>
</table>

For any given slope the corresponding direction angles $\alpha$ and $\gamma$ are setout from the base and the top as shown in Fig 13. The point of intersection of these two lines is the centre of critical circle.

After locating the centre of critical circle the method of slices can be adopted to obtain minimum F.S.
1.15 Critical Slip circle in C-ϕ Soils

In case of c-ϕ soils the procedure for locating critical slip surface is slightly different and is as given below.

![Diagram of critical slip circle in C-ϕ soil]

**Fig 14: Location of critical circle in c-ϕ soil**

1. Locate point O₁, the centre of Fellenius circle
2. Locate point P at 2H below the top surface of the slope and 4.5H from the toe of the slope as shown in Fig. 14.
3. Extend backwards the line PO₁ beyond O₁
4. Construct trial slip circles with centres located on the extended portion of the line PO₁
5. For each of these trial slip circles find the F.S by the method of slices.
6. Plot the F.S for each of these trial slip circles from their respective centres and obtain a curve of factor of safety.
7. Critical slip circle is the one that has a minimum F.S.

1.16 Effective Stress Analysis

When the pore water pressures exist in the embankment due to seepage, sudden drawdown or due to any other reason, then stability should be computed based on effective stress analysis

\[ \sigma = \sigma' + u \quad ; \quad \sigma' = \sigma - u \]
Stability during steady seepage

When seepage occurs at a steady rate through an earth dam or embankment it represents critical condition for the stability of slope.

When seepage occurs pore water pressure (u) develops and this will reduce the effective stress which in turn decreases the shear strength along the failure surface.

The following procedure is adopted to obtain stability

1. Draw the C/S of the slope
2. Draw the potential failure surface
3. Divide the soil mass into slices
4. Calculate the weight W and the corresponding normal and tangential components for all the slices in the usual way

In addition

For the given slope construct flow net (network of equipotential and flow lines) as shown

![Flow Net in a Finite Slope Under Steady Seepage](image)

*Fig. 15: Flow net in a finite slope under steady seepage*

The average pore water pressure (u) at the bottom of the slice is given by the piezometric head ($h_w$) as

$$u = h_w \gamma_w$$
\[ h_w = \text{piezometric head above the base of the slice} \]

The total force due to pore water pressure at the bottom of the slice

\[ U = u \gamma_w \]

Tabulate all the values as shown below

<table>
<thead>
<tr>
<th>Slice No.</th>
<th>Width (m²)</th>
<th>Area</th>
<th>Weight ‘W’ (kN)</th>
<th>Normal component ‘N’ (kN)</th>
<th>Tangential component ‘T’ (kN)</th>
<th>Pore water pressure (u)</th>
<th>Total force due to pore pressure (U)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \Sigma N = \ldots \quad \Sigma T = \ldots \quad \Sigma U = \ldots \]

The F.S is computed as

\[ \text{Factor of Safety, } FS = \frac{(c' r \theta + \tan \phi' \Sigma (N - U))}{\Sigma T} \]

\( c' \) and \( \phi' \) - Shear parameters based on effective stress analysis obtained from drained shear tests.

If the flownet is not constructed then F.S may be computed as

\[ \text{Factor of Safety, } FS = \frac{(c' r \theta + \tan \phi' \Sigma (N'))}{\Sigma T} \]

\( N' = W' \cos \delta = b Z \gamma' \cos \delta \)

\( N' \) = Weight of slice computed from effective unit weight

\( T = W \sin \delta = b Z \gamma_{sat} \sin \delta \) - Weight of slice computed from saturated unit weight.
1.17 Friction Circle Method

This method uses total stress based limit equilibrium approach. In this method the equilibrium of the resultant weight ‘w’, the reaction ‘p’ due to frictional resistance and the cohesive force ‘c’ are considered. The magnitude direction and line of action of ‘w’, the line of action of the reaction force ‘p’ and the cohesive force ‘c’ being known the magnitude of p and c are determined by considering the triangle of forces. The F.S. w. r. t. cohesion and friction is evaluated.

The procedure is as follows:

1. Consider a slope shown in Figure

2. Draw a trial circular slip surface (Arc AC) from the toe as shown with ‘O’ as centre and ‘R’ as radius

3. Find the centroid of the sliding mass ABCA and calculate it’s weight ‘W’
4. For analysis the following 3 forces are considered:
   - The weight $W$ of the sliding soil mass
   - The total reaction $P$ due to frictional resistance
   - The total cohesive force $C$ mobilized along the slip surface

**Force Triangle**

Resultant cohesive force $C$ and its point of application

5. Let the slip circle be considered to be made up of a number of elementary arcs each of length $\Delta L$
6. Let the cohesive force acting along this element opposing the sliding of soil be \( C_m \times \Delta L \). \( C_m \) is the mobilized cohesion.

7. The total cohesive force along the arc AC forms a force polygon.

8. The position of resultant can be obtained by Varignon’s theorem.
9. The cohesive forces $C_m \Delta L$ along the slip circle can be replaced by their resultant $C = C_m \times L_c$ acting parallel to chord AC at a distance $a > R$ from the centre of rotation as shown.

\[
C \ a = \sum C_m \ \Delta L \ \ R \\
C_m \ L_c \ a = C_m \ L \ R \\
a = \frac{L}{L_c} \ R \\
L > L_c \ \therefore \ a > R
\]

Reaction ‘P’ due to Frictional Resistance

10. On mobilization of frictional resistance. Let $P$ be the soil reaction opposing the sliding of soil mass as shown. $P$ is inclined at an angle $\varphi$ to the normal at the point of action as shown.
The line of action of $P$ will pass as tangent to a circle of radius $R \sin \phi$ drawn with ‘O’ as centre called “Friction Circle” or $\phi$- Circle.

The three forces considered for analysis are:

- The weight ‘$W$’ drawn as vertical passing through the centroid of sliding mass (ABCA)
- The resultant cohesive force ‘$C$’ drawn parallel to the chord AC at a distance ‘$a$’ from the centre ‘O’
- The resultant reaction ‘$P$’ passing through the point of intersection of ‘$W$’ and ‘$C$’ and tangential to friction circle
By knowing the magnitude and direction of ‘W’ and the direction and line of action of other forces the force triangle can be completed. Measuring the magnitude of C the F.S. is computed as shown below

The mobilized cohesion \( C_m = \frac{C}{L_c} \)

F.S w.r.t cohesion is given by

\[ F.S = \frac{C_u}{C_m} \]

The minimum F.S is obtained by locating the critical slip circle

**Computation of F.S w.r.t Strength**

1. Assume a trial F.S w.r.t friction as \( F_\varphi \)
2. Draw a friction circle with a reduced radius \( R \sin \varphi_m \)
   
   Where \( \tan \varphi_m = \frac{\tan \varphi_u}{F_\varphi} \)
3. Carry out the friction circle analysis and find the F.S w.r.t cohesion
   
   \[ F_c = \frac{C_u}{C_m} \]
4. If \( F_c = F_\varphi \) its represents F.S w.r.t strength
5. Otherwise choose different \( F_\varphi \) and repeat the procedure till \( F_c = F_\varphi \)

**1.18 Stability Number**

In a slope the component of the self weight (\( \gamma \)) causes instability and the cohesion contributes to stability. The maximum height (\( H_c \)) of a slope is directly proportional to unit cohesion (\( C_u \)) and inversely proportional to unit weight (\( \gamma \)). In addition, \( H_c \) is also related to friction angle (\( \phi_u \)) and slope angle \( \beta \).

This can be expressed as

\[ H_c = \frac{C_u f(\phi_u, \beta)}{\gamma} \]

When the term \( f(\phi_u, \beta) \) is dimensionless then equation above is dimensionally balanced
Taylor (1937) expressed $f(\phi, \beta)$ as a reciprocal of a dimensionless number called “Stability Number” (Sn) popularly called as Taylor’s stability Number.

$$f(\phi, \beta) = \frac{1}{S_n}$$

$$\therefore H_c = \frac{C_u}{\gamma S_n}$$

$$S_n = \frac{C_u}{\gamma H_c}$$

**Problem**

1. Find the Factor of safety against sliding along the interface for the infinite slope shown in Figure. Also find the height $Z$ that will give F.S of 2 against sliding along the interface.

Part - A

$$FS = \frac{\text{Shearing strength}}{\text{Shearing stress}} = \frac{c + \gamma Z \cos^2 \beta \tan \phi}{\gamma Z \sin \beta \cos \beta}$$
Part - B

FS = \frac{10 + (16 X Z^2 \cos^2 25 \tan 15)}{16 X Z^2 \cos 25 \sin 25} = \frac{18.554}{14.89}

FS = 1.24

Part - B

FS = \frac{c + \gamma Z \cos^2 \beta \tan \phi}{\gamma Z \sin \beta \cos \beta}

2 = \frac{10 + (16 X Z) \cos^2 25 \tan 15}{16 X Z \cos 25 \sin 25}

12.25 Z = 10 + 3.52 Z

Z = 1.145 mtrs

2. Dec. 06/Jan.07, Question No: 5.b

An embankment is to be made of a sandy clay, having cohesion of 30 kN/m², Angle of internal friction 20° and a unit weight of 18 kN/m³. The slope and height of embankment are 1.6:1 and 12.0 m respectively. Determine the factor of safety by using the trial circle given in Fig by method of slices

Solution

Given data

Side slope = 1.6:1

Height H = 12 m

Cohesion c = 30 kN/m²
Angle of internal friction $\phi = 20^\circ$

Unit weight $\gamma = 18\text{kN/m}^3$

**Step 1**: Draw the slope to convenient scale.

**Step 2**: From the centre of rotation draw the slip surface.

**Step 3**: Divide the failure plane into 9 slices as shown.

**Step 4**: Measure the average depth ‘Z’ and the breadth ‘B’ of each slice and calculate the areas of each of these slices.

**Step 5**: Calculate the weight of each slices.

$$W = \gamma A$$

**Step 6**: Calculate the angle ‘$\delta$’ for each of the slices.

**Step 7**: Tabulate all the values as shown in the Table and calculate ‘N’ & ‘T’ components.

**Step 8**: Obtain $\Sigma N$ and $\Sigma T$. 
<table>
<thead>
<tr>
<th>Slice No.</th>
<th>Z</th>
<th>B</th>
<th>Area (m²)</th>
<th>Weight (kN/m)</th>
<th>δ (deg)</th>
<th>N = W Cos δ</th>
<th>T = W Sin δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>6.23</td>
<td>112.16</td>
<td>-33</td>
<td>94.06</td>
<td>-61.08</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
<td>24.00</td>
<td>432.00</td>
<td>-20</td>
<td>405.94</td>
<td>-147.75</td>
</tr>
<tr>
<td>3</td>
<td>9.55</td>
<td>4</td>
<td>38.20</td>
<td>687.60</td>
<td>-9</td>
<td>679.13</td>
<td>-107.56</td>
</tr>
</tbody>
</table>
3. Dec. 08/Jan.09, Question No: 5.c (V.T.U)

An embankment is inclined at an angle of 35° and its height is 15 m. the angle of shearing resistance is 15° and the cohesion intercept is 20 kN/m². The unit weight of soil is 18 kN/m³. If the Taylor’s stability number is 0.06, determine the FS with respect to cohesion.

Solution:

Given Data

Stability number $S_n = 0.06$

Slope angle $\beta = 35^\circ$

Height $H = 15$ m

$$\text{Factor of safety, } FS = \frac{cr \theta + \sum N \tan \phi}{\sum T}$$

$$= \frac{(30 \times 21.28 \times 88 \times \frac{\pi}{180} + 5081.41 \times \tan 20^\circ)}{1372.73}$$

$$\text{Factor of safety, } FS = 2.06$$
Cohesion \( c = 20 \text{kN/m}^2 \)

Shearing resistance \( \phi = 15^\circ \)

Unit weight of soil \( \gamma_w = 18 \text{kN/m}^3 \)

\[
\text{Factor of safety, } \text{FS} = \frac{C_u}{S_n \times \gamma_w \times H} = \frac{20}{0.06 \times 18 \times 15} = 1.23
\]

4. Aug -2001, Question No: 5.c (V.T.U)

An embankment is to be constructed with \( c = 20 \text{kN/m}^2, \phi = 20^\circ, \gamma = 18 \text{kN/m}^3, \text{FS} = 1.25 \) and height is 10 m. Estimate side slope required. Taylor’s Stability Numbers are as follows for various slope angles.

<table>
<thead>
<tr>
<th>Slope angle</th>
<th>90</th>
<th>75</th>
<th>60</th>
<th>45</th>
<th>30</th>
<th>20</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_n )</td>
<td>0.182</td>
<td>0.134</td>
<td>0.097</td>
<td>0.062</td>
<td>0.025</td>
<td>0.005</td>
<td>0</td>
</tr>
</tbody>
</table>

Also find the factor of safety, if the slope is 1V: 2H given \( \phi = 20^\circ \)

Solution:

Given data:

Height \( H = 10 \text{ m} \)

Cohesion \( c = 20 \text{kN/m}^2 \)

Shearing resistance \( \phi = 20^\circ \)

Unit weight of soil \( \gamma_w = 18 \text{kN/m}^3 \)

FS = 1.25
Stability of Earth Slopes

Part-A

Factor of safety, \( FS = \frac{C}{S_n \times \gamma_w \times H} \)

\textbf{Stability number, } S_n = \frac{C}{FS \times \gamma_w \times H}

\( S_n = \frac{20}{1.25 \times 18 \times 10} \)

\textbf{Stability number, } S_n = 0.088

Therefore Slope angle \( \beta = 56.14° \)

Part-B

Slope angle \( \beta = \tan^{-1} \left( \frac{1}{2} \right) \)

\( \beta = 26.5° \)

For \( \beta = 26.5° \), Taylor's Stability Number \( S_n = 0.018 \)

Factor of safety, \( FS = \frac{C}{S_n \times \gamma_w \times H} \)

\( FS = \frac{20}{0.018 \times 18 \times 10} \)

\( FS = 6.17 \)

5. July -2007, Question No: 5.b (V.T.U)

A 5 m deep canal has side slopes of 1:1. The properties of soil are \( c_u = 20 \) kN/m², \( \phi_u = 10° \), \( c = 0.8 \) and \( G = 2.8 \). If Taylor’s stability number is 0.108, determine the factor of safety with respect to cohesion when the canal runs full. Also find the same in case of sudden drawdown, if Taylor’s stability number for this condition is 0.137.

Solution:

Given data

Stability number \( S_n = 0.108 \)
Side slope = 1:1
Height H = 5 m
Cohesion $c_u = 20$ kN/m$^2$
Shearing resistance $\phi_u = 10^\circ$
Void ratio $e = 0.8$
Specific gravity $G = 2.8$

$$\gamma_{sat} = \frac{G + e}{1 + e} \gamma_w$$

$$\gamma_{sat} = \frac{2.8 + 0.8}{1 + 0.8} \times 9.81$$

$$\gamma_{sat} = 19.62 \text{ kN/m}^3$$

$$\gamma' = 19.62 - 9.81 = 9.81 \text{ kN/m}^3$$

$$i = 45^\circ, \phi = 10^\circ$$

(1) Submerged case or canal runs full

$$i = 45^\circ, \phi = 10^\circ, S_o = 0.108$$

Factor of safety, $FS = \frac{C}{S_n \times \gamma' \times H}$

Factor of safety, $FS = \frac{20}{0.108 \times 9.81 \times 3}$

Factor of safety, $FS = 3.77$
(II) Drawdown Case

\[ S_n = 0.137 \]

Factor of safety, \( FS = \frac{C}{S_n \times \gamma \times H} \)

Factor of safety, \( FS = \frac{20}{0.137 \times 19.62 \times 5} \)

Factor of safety, \( FS = 1.49 \)

6. July/August -2002, Question No: 5.c (V.T.U)

An embankment is inclined at an angle of 35° and its height is 15 m. The unit weight of soil is 18.0 kN/m³. If the Taylor’s stability number is 0.06, find the factor of safety with respect to cohesion.

Solution

Given data

Stability number \( S_n = 0.06 \)

Slope angle \( \beta = 35° \)

Height \( H = 15 \) m

Unit weight of soil \( \gamma = 18 \) kN/m³

Factor of safety, \( FS = \frac{C}{S_n \times \gamma \times H} \)

Factor of safety, \( FS = \frac{C}{0.06 \times 18 \times 15} \)

Factor of safety, \( FS = 0.062C \)
7. July/August -2002, Question No: 5.c (V.T.U)

An embankment of 10m height is constructed in a soil having $c = 0.02$ N/mm$^2$, $\phi = 20^\circ$ and $\gamma = 6$ kN/m$^3$. Find the Factor of safety with respect to cohesion and also the critical height of the embankment. Assume Stability number $= 0.05$.

Solution

Given Data

Stability number $S_n = 0.05$
Cohesion $c = 0.02$ N/mm$^2$
Height $H = 10$ m
Unit weight of soil $\gamma = 6$ kN/m$^3$

Factor of safety, $FS = \frac{C}{S_n \times \gamma \times H}$

Factor of safety, $FS = \frac{0.02}{1000 \times (1000 \times 1000)} \times \frac{1}{0.05 \times 6 \times 10}$

Factor of safety, $FS = 6.67$

Critical Height, $H_c = FS \times H$

Critical Height, $H_c = 6.67 \times 10$

Critical Height, $H_c = 66.7$ m