Subsurface Stresses
(Stresses in soils)

ENB371: Geotechnical Engineering 2
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  • Infinitely loaded area
  • Point load (concentrated load)
  • Circular loaded area
  • Strip load
  • Rectangular loaded area
Introduction -1

At a point within a soil mass, stresses will be developed as a result of the soil lying above the point (Geostatic stress) and by any structural or other loading imposed onto that soil mass.

The subsurface stress at a point is affected by ground water table if it extends to an elevation above the point.

Ground level

Unit weight

\[ \text{Unit weight} = \gamma \]

\[ h \]

Total stress = \[ \sigma = \gamma h \]

Total stress = \[ \sigma = \gamma h + \Delta \sigma \]
Geostatic stresses at a point in soil

\[ K_0 \approx 1 - \sin \phi \]

\( K_0 = \) Coefficient of earth pressure at rest, \( \phi = \) friction angle

Soil unit weight = \( \gamma_s \)

\[ K_0 = \frac{\sigma'_h}{\sigma'_v} \]

\[ \sigma_v = z * \gamma_s \]

\[ \sigma'_h = K_0 * \sigma'_v \]
Introduction

Subsurface stresses in road pavements and airport runways are increased by wheel load on the surface.
Subsurface stresses in soils are increased by foundation loads.
Introduction - 5

Embankments and landfills cause to increase subsoil stresses
Subsoil stress increase -1

It is required to estimate the stress increase in the soil due to the applied loads on the surface. The estimated subsoil stress increase is used:

- to estimate settlement of foundation
- to check the bearing capacity of the foundation
Subsoil stress increase

The calculation of subsurface stress increase due to the following surface loading conditions will be discussed.

- Infinitely loaded area
- Point load (concentrated load)
- Circular loaded area
- Strip loading
- Rectangular loaded area

Though the surface loading is caused to increase both vertical and horizontal stresses in soils, only the vertical stress increase is discussed as it is the major stress component for most practical design problems.
The surface loading area is much larger than the depth of a point where vertical stress increment ($\Delta \sigma$) is calculated (e.g. landfills, preloading by soil deposition).
The vertical subsurface stress increment at any depth below the infinitely loaded area is considered to be the same as the surface stress due to external load (filling material).

\[ \Delta \sigma_1 = \Delta \sigma_2 = \Delta \sigma = q \]
Finitely loaded area

If the surface loading area is finite (point, circular, strip, rectangular, square), the vertical stress increment in the sub-soil decreases with increase in the depth and the distance form the surface loading area.

Methods have been developed to estimate the vertical stress increment in sub-soil considering the shape of the surface loading area.
Subsurface stress increment due to a point load

- For homogeneous (properties are the same everywhere), and isotropic (properties are the same in all direction) soils

- Boussinesq (1885) (French mathematician) developed the following equation to calculate vertical stress increment ($\Delta \sigma$) in soils due to point load on the surface:

$$\Delta \sigma = \frac{3Q}{2\pi z^2} \left[ \frac{1}{1 + \left( \frac{r}{z} \right)^2} \right]^{5/2}$$

Then,

$$\Delta \sigma = \frac{Q}{Z^2} I_p$$

**Influence factor, $I_p$**

$$I_p = \frac{3}{2\pi} \left[ \frac{1}{1 + (r/z)^2} \right]^{5/2}$$

$I_p$ can be obtained numerically or using table where $I_p$ is given in terms of $r/z$
Subsurface stress increment due to a point load

Influence factor, $l_p$, for vertical stress increment due to point load

<table>
<thead>
<tr>
<th>$r/z$</th>
<th>$l_p$</th>
<th>$r/z$</th>
<th>$l_p$</th>
<th>$r/z$</th>
<th>$l_p$</th>
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Example -1 (1)

In road pavement design, the standard vehicle axel is defined as an axel with two single wheels as shown below.

For a particular vehicle group, the standard axel load \((P)\) is given as 80 kN and the distance between two wheels \((L)\) is 1.8 m. What is the vertical stress increment in the sub-grade at 4 m depth directly under a wheel if this axel is running. (consider wheel load as a point load)
Find: $\Delta \sigma$

$\Delta \sigma = \Delta \sigma_A + \Delta \sigma_B$

$\Delta \sigma = \frac{Q}{z^2} I_p$

\[
(I_p)_B = \frac{3}{2\pi} \left\{ \frac{1}{1 + (r/z)^2} \right\}^{5/2} = \frac{3}{2\pi} \left\{ \frac{1}{1 + (1.8/4)^2} \right\}^{5/2} = 0.301
\]

\[
(I_p)_A = \frac{3}{2\pi} \left\{ \frac{1}{1 + (r/z)^2} \right\}^{5/2} = \frac{3}{2\pi} \left\{ \frac{1}{1 + (0/4)^2} \right\}^{5/2} = 0.477
\]

Therefore,

\[
\Delta \sigma = \frac{Q_A}{z^2} (I_p)_A + \frac{Q_B}{z^2} (I_p)_B = \frac{40}{4^2} \times 0.477 + \frac{40}{4^2} \times 0.301 = 1.945 \text{ kPa}
\]
Subsurface stress increment due to circular loaded area -1
Subsurface stress increment due to circular loaded area -2

Considering force on the small element (black) And applying Boussinesq equation, the stress Increase at A caused by this load

\[ d(\Delta \sigma) = \frac{3(q_0 r \ d\theta \ dr)}{2\pi z^2 \left[ 1 + \left( \frac{r}{z} \right)^2 \right]^{5/2}} \]

By integration of Boussinesq solution over the whole circular area, the vertical stress increment at depth \( z \) under the centre (at A)

\[ \Delta \sigma = q_0 \left\{ 1 - \frac{1}{\left[ 1 + \left( \frac{B}{2z} \right)^2 \right]^{3/2}} \right\} = q_0 I_c \]

The influence factor \( I_c \) can be calculated numerically or can be obtained from a table
Subsurface stress increment due to circular loaded area

**Table 5.1** Variation of $\Delta \sigma / q_o$ for a Uniformly Loaded Flexible Circular Area

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<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
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<td>1.000</td>
<td>1.000</td>
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Similar integration can be performed to obtain the vertical stress increase at $A'$, located a distance $r$ from the centre of the loaded area at a depth $z$. This table can be used to calculate $I_c$ corresponding to $A'$. 
Subsurface stress increment due to strip loading -1
Vertical stress increment due to strip area carrying uniform pressure

- Soil is homogeneous, isotropic

- The stress increment at point X ($\Delta \sigma_z$) due to a uniform pressure $q$ on a strip area of width $B$ and infinite length is given by in terms of $\alpha$ and $\beta$

$$\Delta \sigma = \frac{q}{\pi} \left\{ \alpha + \sin \alpha \cos(\alpha + 2\beta) \right\}$$
Vertical stress increment due to strip area carrying increasing pressure

- Soil is homogeneous, isotropic

- The stress increment at point $X$ ($\Delta \sigma_z$) due to increasing pressure from zero to $q$ on a strip area of width $B$ and infinite length is given by in terms of $\alpha$ and $\beta$ the lengths $R_1$ and $R_2$.

\[
\Delta \sigma_z = \frac{q}{\pi} \left\{ \frac{X}{B} \alpha - \frac{1}{2} \sin 2\beta \right\}
\]
Example - 2

A strip footing 2 m wide carries a uniform pressure of 250 kPa on the surface of a deposit of sand. Water table is at the surface and the saturated unit weight of sand is 20 kN/m$^3$. Determine the effective vertical stress at a point 3 m below the centre of the footing before and after the application of the surface pressure.

**Before loading**

Total vertical stress at 3 m depth, $\sigma_v$

$$\sigma_v = \gamma_{sat} z = 20 \times 3 = 60 \text{ kPa}$$

Pore water pressure at 3 m depth, $u$

$$u = \gamma_w z = 9.8 \times 3 = 29.4 \text{ kPa}$$

Effective vertical stress at 3 m depth, $\sigma'_v$

$$\sigma'_v = \sigma_v - u = 60 - 29.4 = 30.6 \text{ kPa}$$
Strip area carrying uniform pressure - Example

After loading

\[ B = 2 \text{ m}, \ q = 250 \text{ kPa}, \ z = 3 \text{ m}, \]

At the centre of the strip, \( \beta = -\alpha/2 \)

Therefore, \( \cos(\alpha + 2\beta) = 1 \)

\[
\tan\left(\frac{\alpha}{2}\right) = \frac{1}{3}
\]

\[
\alpha = 2 \tan^{-1}\left(\frac{1}{3}\right) = 36^0\ 52' = 0.643 \text{ radians}
\]

\[
\sin \alpha = 0.600
\]

The increase in stress at 3 m depth due to applied load

\[
\Delta \sigma = \frac{q}{\pi} \left\{ \alpha + \sin \alpha \cos(\alpha + 2\beta) \right\}
\]

\[
\Delta \sigma_z = \frac{q}{\pi} \left( \alpha + \sin \alpha \right) = \frac{250}{\pi} \left( 0.643 + 0.600 \right) = 99.0 \text{ kPa}
\]

Hence, total effective vertical stress = 30.6 + 99.0 = 129.6 kPa
Subsurface stress increment due to rectangularly/squarely loaded area -1
Vertical stress increment due to uniformly loaded rectangular area -1

Considering force on the small element (black) and applying Boussinesq equation, the stress increase at A caused by this load.

\[ d(\Delta \sigma) = \frac{3q_0 (dx \; dy)z^3}{2\pi \left[ x^2 + y^2 + z^2 \right]^{5/2}} \]

By integration of Boussinesq solution over complete area, the vertical stress increment at depth \( z \) under the centre (at A)

\[ \Delta \sigma = \int_{y=0}^{L} \int_{x=0}^{B} \frac{3q_0 (dx \; dy)z^3}{2\pi \left[ x^2 + y^2 + z^2 \right]^{5/2}} = q_0 I_r \]

\[ \Delta \sigma_z = \frac{3Q}{2\pi \xi^2} \left[ \frac{1}{1 + \left( \frac{r}{z} \right)^2} \right]^{5/2} \]
Vertical stress increment due to uniformly loaded rectangular area -2

- The stresses increase under a corner of a uniformly loaded flexible rectangular area:

\[ \Delta \sigma = q_0 I_r \]

Influence factor for rectangular footing, \( I_r \)

\[ I_r = \frac{1}{4\pi} \left[ \frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2-m^2n^2+1} \cdot \frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1} \frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2-m^2n^2+1} \right] \]

- Define \( m = B/z \) and \( n = L/z \)
- Solution by numerically or tables
Influence factor, $I_r$, for vertical stress increment due to uniformly loaded rectangular area -1

### Table 5.2 Variation of Influence Value $I_r$ [Eq. (5.6)]

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<thead>
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<th>0.9</th>
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Influence factor, $I_r$, for vertical stress increment due to uniformly loaded rectangular area -2

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*After Newmark, 1935.*
The vertical stress increment at a depth $z$ below point $O$,}

$$
\Delta \sigma_z = q_0 \left( I_1 + I_2 + I_3 + I_4 \right)
$$
Example -3

Determine the increase in stress at A and A’ below a rectangular area

At the centre of rectangular or square area,

\[
\Delta \sigma = q_0 (I_1 + I_2 + I_3 + I_4)
\]

\[
I_1 = I_2 = I_3 = I_4
\]

Then,

\[
\Delta \sigma = 4q_0 (I)
\]

\[
B = L = 1.5 \, m
\]

\[
m_A = n_A = \frac{B \text{ or } L}{z_A} = \frac{1.5}{3} = 0.5, \, I_A = 0.084
\]

\[
m_{A'} = n_{A'} = \frac{B \text{ or } L}{z_{A'}} = \frac{1.5}{5} = 0.3, \, I_{A'} = 0.037
\]

\[
\Delta \sigma_A = 4 \times 100 \times 0.084 = 33.6 \, kPa
\]

\[
\Delta \sigma_{A'} = 4 \times 100 \times 0.037 = 14.8 \, kPa
\]
Example – 4 (1)

A rectangular foundation 6 X 3 m carries a uniform pressure of 300 kPa near the surface of a soil mass. Determine the vertical stress at a depth of 3 m below point A on the centre line 1.5 m outside the long edge of the foundation.

Hint: use the principle of superposition
Example - 4 (2)

Using the principle of superposition the problem is dealt as shown below

For the two rectangles (1) carrying a positive pressure of 300 kPa,

$B = 3.0 \text{ m}, \quad L = 4.5 \text{ m}, \quad m = B/z = 3.0/3.0 = 1.0, \quad n = L/z = 4.5/3.0 = 1.5$

Therefore, $I_r = 0.193$ (from the table)

For the two rectangles (2) carrying a negative pressure of 300 kPa,

$B = 1.5 \text{ m}, \quad L = 3.0 \text{ m}, \quad m = B/z = 1.5/3.0 = 0.5, \quad n = L/z = 3.0/3.0 = 1.0$

Therefore, $I_r = 0.120$ (from the table)

Hence, vertical stress increase at A (a depth of 3 m),

\[
(\Delta \sigma_z)_A = 2(q_0I_r)_1 + 2(q_0I_r)_2 = (2 \times 300 \times 0.193) - (2 \times 300 \times 0.120) = 44 \text{ kPa}
\]
Newmark (1942) constructed influence chart, based on the Boussinesq solution to determine the vertical stress increase at any point below an area of any shape carrying uniform pressure.

- Chart consists of influence areas which has a influence value of 0.005 per unit pressure
- The loaded area is drawn on tracing paper to a scale such that the length of the scale line on the chart is equal to the depth $z$
- Position the loaded area on the chart such that the point at which the vertical stress required is at the centre of the chart.
- The count the number of influence areas covered by the scale drawing, $N$

Then, vertical stress increase at $z$

$$\Delta \sigma_z = 0.005 q_o N$$
Influence chart for vertical stress increase -2

- Use of Newmark’s chart to calculate stress increment at A, (at a depth of 3.0 m)

\[ Z = 3.0 \text{ m} \]
\[ q_0 = 300 \text{ kPa} \]

Then, vertical stress increase at 3 m below point A
\[ \Delta \sigma_z = 0.005 \ q_0 \ N = 0.005 \times 300 \times 30 = 45 \text{ kPa} \]
Approximate method to determine the stress subsurface stress increment (60° approximation)

According to this method, the increase in stress at depth $z$ is

$$
\Delta \sigma = \frac{q_0 \times B \times L}{(B + z)(L + z)}
$$

Figure 5.5  2:1 method of finding stress increase under a foundation
Example – 5 (1)

Compare the stress increase occurring 2m below the centre of a 3m by 3m square foundation imposing a bearing pressure of 145 kN/m² when:

(i) The Boussinesq stress distribution is assumed
(ii) The 60° approximation is assumed
Example – 5 (2)

(i) The Boussinesq stress distribution is assumed

\[ \Delta \sigma = q_0 (I_1 + I_2 + I_3 + I_4) \]

When the centre is considered,

\[ I_1 = I_2 = I_3 = I_4 \]

Then,

\[ \Delta \sigma = 4q_0 (I) \]

\[ q_0 = 145 \text{ kN/m}^2 \]

\[ B' = L' = 1.5 \text{ m} \]

\[ m = n = \frac{B' \text{ or } L'}{z} = \frac{1.5}{2} = 0.75, I = 0.1367 \]

\[ \Delta \sigma_{2m} = 4 \times 145 \times 0.1367 = 79.3 \text{ kPa} \]
Example – 5 (2)

(ii) The $60^\circ$ approximation is assumed

\[ B = L = 3.0 \text{ m} \]

\[ q_0 = 145 \text{ kN} / \text{m}^2 \]

\[ z = 2 \text{ m} \]

At 2m depth,

\[ \Delta \sigma = \frac{q_0 \times B \times L}{(B + z)(L + z)} \]

\[ \Delta \sigma = \frac{145 \times 3 \times 3}{(3 + 2)(3 + 2)} = \frac{145 \times 9}{5 \times 5} = 52.2 \text{ kN} / \text{m}^2 \]
END