Introduction

Some physical, mathematical and geologic background needed to be able to quantify groundwater movement

1. a review of energy balance, to allow quantification of the driving force for fluid flow
2. a brief review of quantitative sedimentology, to allow for description of (sedimentary) rocks, and their ability to transmit fluids
Work and Energy

- Recall that work is equal to the product of the net force exerted and the distance over which the force moves

\[ W = F \cdot D \]  

\[ \text{Work } \left( \frac{ML^2}{T^2} \right) \quad \text{Force } \left( \frac{ML}{T^2} \right) \quad \text{Distance } (L) \]  

- Work is done when moving any object in a gravitational field (e.g. water), pressurizing a gas or fluid, etc.

- Recall that force is the mass times the acceleration of a body (Newton’s 2nd Law)

\[ F = m \cdot a \]  

\[ \text{mass } (M) \quad \text{acceleration } \frac{L}{T^2} \]
Units for Work and Energy

Table 1: Comparison of SI and English units for work and energy. Note especially that kilograms and pounds are not equivalent parameters, despite common usage (OK as long as $\ddot{g}$ is constant).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SI</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>m</td>
<td>ft</td>
</tr>
<tr>
<td>Time</td>
<td>sec</td>
<td>sec</td>
</tr>
<tr>
<td>Mass</td>
<td>kg</td>
<td>slug</td>
</tr>
<tr>
<td>Force</td>
<td>$\frac{kg \cdot m}{sec^2}$ = Newton</td>
<td>pound</td>
</tr>
<tr>
<td>Pressure</td>
<td>Pascal = $\frac{Newton}{m^2}$</td>
<td>$\frac{lb}{ft^2}$</td>
</tr>
</tbody>
</table>
Porosity

- The sub-surface void space that holds groundwater is **porosity**
- The basic definition of porosity is the ratio of void space (volume) to total volume

\[
\begin{align*}
    n\% &= 100 \cdot \frac{V_v}{V} = \frac{\text{void volume}}{\text{total volume}} \quad (3)
\end{align*}
\]

- In the laboratory soil/rock (**effective**) porosity is most simply determined by:
  1. measuring the original volume of sample
  2. drying the sample to remove unbound water
  3. submerging dried sample in a known volume of water until saturated
  4. the void volume \( (V_v) \) is the difference between the original water volume minus that remaining after the saturated sample is removed
Porosity (cont.)

- Total porosity is generally computed using:

\[
  n\% = 100 \cdot \left[1 - \frac{\rho_b}{\rho_d}\right]
\]  

- \(\rho_b\) is the bulk density of the solids (i.e. dried mass over original volume, often assumed to be 2650 \(\frac{\text{kg}}{\text{m}^3}\))

- \(\rho_d\) is the particle density (dried mass over particle volume, determined by water displacement test)

- these two measures can differ dramatically depending on rock formation
Classification of Sediments

- Soils and lithified sediments are initially classified by grain size (Figs. 1–2)
- Grain size distribution is measured by stacked sieves for coarse fraction, hydrometer for fine fraction (relies on Stoke’s Law)
- Most environmental studies will soils, standard soil classification is the Unified Soil Classification System (Fig. 5), and Wikipedia summary
- Table 3.3 of Fetter (2001) gives useful field tests that help distinguish the ratio of clay to silt (Fig. 6). Note that organic content is generally determined by reaction with hydrogen peroxide.
- Grain size distribution ultimately determines the permeability of the rock. This is quantified using diagrams (Fig. 3), and by
  - effective grain size - the size corresponding to the 10% line on (Fig. 3)
Classification of Sediments (cont.)

- **uniformity coefficient** - a direct measure of the sorting given by

\[ C_u = \frac{d_{60}}{d_{10}} \]

- For modeling purposes, porosity is often estimated from tabulated averages (Fig. 4)
**Scales for Grain Size**

**Figure 1:** Commonly applied grain size scales for field characterization of sediments, after Dietrich et al. (Datasheet 17-2, 1982).
Estimating Grain Size

Figure 2: Grain size estimation chart, after Dietrich et al. (Datasheet 16-1, 1982).
Grain Size Distribution Curves

Figure 3: Grain Size Distribution Curves, after (Fig. 3.4, Fetter, 2001).
Typical Porosity Ranges

**Table 3.4** Porosity Ranges for Sediments

<table>
<thead>
<tr>
<th>Material</th>
<th>Porosity Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well-sorted sand or gravel</td>
<td>25–50%</td>
</tr>
<tr>
<td>Sand and gravel, mixed</td>
<td>20–35%</td>
</tr>
<tr>
<td>Glacial till</td>
<td>10–20%</td>
</tr>
<tr>
<td>Silt</td>
<td>35–50%</td>
</tr>
<tr>
<td>Clay</td>
<td>33–60%</td>
</tr>
</tbody>
</table>

Based on Meinzer (1923a); Davis (1969); Cohen (1965); and MacCary and Lambert (1962).

**Figure 4:** Typical porosity ranges for sediments, after (Table 3.4, Fetter, 2001). The form and electric charge on clay particles causes poor packing, making porosity of clays high.
Figure 5: Unified Soil Classification System, after Dietrich et al. (Datasheet 26.1, 1982).
Figure 6: ASTM Standard D2488 for Soil Description, after (Table 3.3, Fetter, 2001). “Toughness” is the consistency of the soil near the plastic limit.
Geotechnical Properties of Soil

Often want to know soil strength as affected by water content. In general these are termed the Atterberg Limits, which seek to define the boundaries between liquid, plastic, semi-solid and solid states.

- **Liquid Limit (LL):**
  - The water content corresponding to an arbitrary limit between the liquid and plastic states of consistence of a soil.
  - The water content at which a pat of soil, cut by a standard-sized groove, will flow together for a distance of 12 mm under the impact of 25 blows in a standard liquid-limit apparatus (see also lab class procedure).

- **Plastic Limit (PL):**
  - The water content corresponding to an arbitrary limit between the plastic and the semisolid states of consistence of a soil.
  - The water content at which a soil will just begin to crumble when rolled into a thread approximately 3 mm in diameter.
Plasticity Index (PI): is given by $PI = LL - PL$, highly plastic (high PI) soils tend to be clay rich and expansive (all that water has to go somewhere!)

Note water (moisture) content is obtained by measuring original sample volume ($V_{total}$) and mass ($M_{wet}$), then drying it and obtaining dry mass $M_{dry}$. Then volumetric moisture content (saturation)

$$\theta = \frac{(M_{wet} - M_{dry})}{\rho_{water} \frac{V_{total}}{V_{water}}} = \frac{V_{water}}{V_{total}}$$

(5)
Geotechnical Origin of Group Names

Figure 7: USCS Group Names vs. geotechnical properties. Silty soils plot to lower left, clay-rich soils to upper right. “A” line separates inorganic clays above from organic clays and silty soils. After British Standards Institute Standard for Site Investigations (BS 5930).
Mineralogical Origin of Group Names

Figure 8: Mineralogic controls on geotechnical properties, see various online summaries.
Figure 9: Unified soil classification system chart, from Virginia DOT.
Modern quantitative observations began with Henri Darcy in 1856, analyzing flow in sand-filled pipes (filters) for a city fountain (central water supply).

Head: Darcy noted that discharge (water mass per cross-sectional area per time) increased with increasing height difference between ends of the pipe. We’ll call this “head difference” or hydraulic gradient.

Hydraulic Conductivity: also noted differing linear relationships between hydraulic gradient and discharge depending on the material (Fig. 10). We’ll call the slope of these lines Hydraulic Conductivity, a material property of the aquifer.
Darcy Results

Figure 10: Darcy experimental results, after (Fig. 3.13, Fetter, 2001).

Hydraulic Conductivity

- depends on both aquifer (intrinsic permeability) and fluid (density and viscosity) properties
- can be related to mean grain size in sediments by the formula (Shepherd method, 1989)
  \[ K = C \cdot d_{50}^j \]
  where \(1 \geq j \geq 2\) (Fig. 11)
- Tabulated ranges are often used in modeling and initial analysis of well testing (Fig. 12)
- note the Hazen method (1911) is also popular, and uses the formula
  \[ K = C \cdot d_{10}^2 \]
Conductivity vs. Mean Grain Size

*Figure 11: Hydraulic Conductivity vs. Mean Grain Size, after (Fig. 3.15, Fetter, 2001). “Textual maturity” essentially refers to sorting/uniformity coefficient.*
### Table 3.7 Ranges of Intrinsic Permeabilities and Hydraulic Conductivities for Unconsolidated Sediments

<table>
<thead>
<tr>
<th>Material</th>
<th>Intrinsic Permeability (darcys)</th>
<th>Hydraulic Conductivity (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>$10^{-6} - 10^{-3}$</td>
<td>$10^{-9} - 10^{-6}$</td>
</tr>
<tr>
<td>Silt, sandy silts, clayey sands, till</td>
<td>$10^{-3} - 10^{-1}$</td>
<td>$10^{-6} - 10^{-4}$</td>
</tr>
<tr>
<td>Silty sands, fine sands</td>
<td>$10^{-2} - 1$</td>
<td>$10^{-5} - 10^{-3}$</td>
</tr>
<tr>
<td>Well-sorted sands, glacial outwash</td>
<td>$1 - 10^{2}$</td>
<td>$10^{-3} - 10^{-1}$</td>
</tr>
<tr>
<td>Well-sorted gravel</td>
<td>$10 - 10^{3}$</td>
<td>$10^{-2} - 1$</td>
</tr>
</tbody>
</table>

**Figure 12:** Typical Values of Conductivity and Permeability, after (Tbl. 3.7, Fetter, 2001). See also lithology correlation.
Groundwater Features

Water in the ground is found in three general zones (Fig. 13):

- **Vadose, or unsaturated zone** (saturation < 1, pore pressure $\Psi < P_{\text{atmospheric}}$)

- **Saturated zone** (saturation = 1, pore pressure $\Psi > P_{\text{atmospheric}}$

- **Capillary zone**, lies above the water table (the line at which $\Psi = P_{\text{atmospheric}}$, saturation = 1, $\Psi < P_{\text{atmospheric}}$)
Groundwater Zones

Figure 13: Groundwater zones, after (Fig. 3.18, Fetter, 2001).
Water Table Features

- A sloping water table indicates water is flowing (Fig. 14)
- Groundwater discharges at topographic (and water table) low-spots
- The water table generally has the same shape as the topography, modified by the location of water source/sinks and distribution of permeability (see Introductory Lecture Notes)
- Water typically flows away from topo highs and toward topo lows
Example Potentiometric Maps

Figure 14: Example Potentiometric Maps, after (Fig. 3.24, Fetter, 2001).