How to Design a Singly Reinforced Concrete Beam

Time Required: 45 minutes
Materials:
- Engineering Paper
- Calculator
- Pencil
- Straight Edge

Design For Flexural Limit State

Goal: \( \Phi M_n > M_u \)
Strength Reduction Factor * Nominal Moment Strength > Required Strength

When designing a singly reinforced concrete beam, the flexural limit state is used to determine when the beam will fail. The flexural limit state is found by multiplying the nominal moment strength \( (M_n) \) by a factor of safety \( (\Phi) \). For singly reinforced beams that fail in tension, the strength reduction factor \( (\Phi) \) will always be .9. This reduction factor is used for safety reasons when designing beams because of variability in material strength and construction methods. As defined by the American Concrete Institute (ACI) the nominal moment strength is reached when the extreme concrete fiber crushes. This is when the strain equals .003. The equations below have been derived from this concept.

Note: Designing a beam is not always a linear process and it may take several tries to satisfy the design requirement. It is not uncommon to repeat this process 2-3 times before satisfying the strength requirements.

1. **Structurally Analyze the Beam**
   The first step for designing a beam is to analysis the load which the beam will be carrying. Beam loads are typically simplified as a uniformed distributed load. Use your knowledge of statics to calculate the shear forces and moments. Concrete beams are designed to fail in flexure, so the maximum moment \( (M_u) \) can be found from the moment diagram.
For this example, we will use a span \((L)\) of 20 feet and a uniformly distributed load \((w)\) of 1000 lbs/ft.

![Diagram of beam with span and load](image)

\[ w = 1000 \text{lb/ft} \]

\[ L = 20' \]

Use the equation below to calculated \(\mu\):

\[
\mu = \frac{w(L)^2}{8} = \frac{1,000 \times 20^2}{8} = 500,000 \text{ lb-ft}
\]

2. **Specify the Compression Strength of the Concrete**
   The engineer must make educated assumptions based on what the beam is supporting in order to specify the concrete’s strength. The heavier the load, the high strength concrete will be required. Most beams are designed using normal concrete. Normal concrete compression strength ranges from 3,000 – 7,000 psi. High strength concrete has compression strengths from 7,000 – 15,000 psi.

   *For this example, we will use concrete with a compression strength \((f'_c)\) of 4,000 psi.*

3. **Specify the Tensile Strength of the Steel Reinforcement**
   For a typical concrete beam the engineer will specify ASTM A615 steel which has a yield strength \((f_y)\) of 60,000 psi. The tensile strength of steel ranges from 40,000 – 120,000 psi. If the beam you are designing has a high load or you want to use less steel, specify a higher tensile strength steel.

   *For this example, we will use steel with a tensile strength \((f_y)\) of 60,000 psi.*

4. **Determine the Cross-Sectional Dimensions of the Beam**
   The size of the beam’s cross-sectional must fit the constraints of the structure you are designing. In a building, a beam cannot have a large depth because of ceiling space it will occupy. However, for a structure like a bridge, a beam may have a depth of several feet because such space limitations do not apply.

   *For this example, we will assume we are designing a beam for a bridge and use a depth \((h)\) of 24 inches and a width \((b)\) of 24 inches.*
5. **Calculate the Area of Steel Required**

The area of steel (As) required in a singly reinforced concrete beam is calculated with the equation:

\[
As > \frac{Mu}{\Phi(fy)(Jd)} > \frac{500,000 \times (\frac{12}{1})}{.9 \times 60,000 \times 18.8} > 5.8 \text{ inches}^2
\]

- \(Mu\) = maximum moment found in step 1
- \(\Phi = .9\) for tension controlled beams
- \(fy\) = tensile strength of the steel specified in step 3
- \(Jd = .875(h-2.5)\). \(h\) is determined in step 4

6. **Select the Area of Steel Provided**

The area of steel provided must be greater than the area of steel required that is calculated in the previous step. Using the steel reinforcement table below, select a dimension size that will require 3 to 6 bars. If you select a bar size that requires more than 6 bars for the required area, reselect a larger bar size. It is difficult in construction to fit more than 6 bars in a narrow beam.

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**For this example, we require 5.8 inches\(^2\) of steel and will use 6 reinforcement bars.**

**Therefore, we will divide 5.8 inches\(^2\) by 6 bars and get .96 inches\(^2\) per bar. Using the chart we must select the bar that is the closest to .96 inches\(^2\) but larger.**

**For this beam we will be using 6 #9 bars which will provide 6 inches\(^2\) of steel.**

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**Note:** The area of steel provided must be slightly larger than the area of steel required.
7. **Calculate the Area of the Compression Block**
   The compression block is the section of the beam which is in compression, and for a beam with a positive bending moment, this area is the top part of the beam. Use the following equation to calculate the area in compression (a):
   
   \[
   a = \frac{(As)(fy)}{.85(f'c)(b)} = \frac{6\times60,000}{.85\times4,000\times24} = 4.41 \text{ inches}
   \]
   
   As = area of steel provided found in step 6
   fy = tensile strength of the steel specified in step 3
   b = width found in step 4
   f’c = compression strength of the concrete specified in step 2

8. **Calculate the Nominal Moment Strength**
   Use the following equation to calculate the nominal moment strength:
   
   \[
   Mn = (As)(fy)[h-2.5-\left(\frac{a}{2}\right)] = 6\times60,000[24-2.5-\left(\frac{4.41}{2}\right)] \left(\frac{1'}{12}\right) = 578,850 \text{ lb-ft}
   \]

9. **Verify that \( \Phi Mn > Mu \)**
   Now that the nominal moment strength (Mn) of the beam has been calculated, we must verify that it is greater than the ultimate moment strength (Mu) that was found from structural analysis in step 1. Plug the values into the equation below:
   
   \[
   \Phi Mn > Mu \quad \Rightarrow \quad (.9)\times578,850 = 530,000
   \]
   
   530,000 > 500,000 \( \Rightarrow \) Verified

   **Note:** If this inequality is not satisfied we must go back to step 2 and increase the strength of the concrete, step 3 and increase the tensile strength of the steel, or step 4 and make the beam’s cross-section larger. If you go back, each step must be recalculated in sequence.

10. **Sketch the Beam on Engineering Paper**
    Draw the outline of the beam to a scale of your choice. For this example we will use a scale of \( \frac{3}{2}'' = 1' \). Be sure to label the height and width of the cross-section, length of the beam, and call out the number of bars used and bar#. The steel reinforcement will always be 2.5 inches from the bottom of the beam when using this method of design.
Appendix: Notation

a = depth of equivalent stress block
As = area of steel reinforcement
b = width of the compression face of the member
f’c = specified compressive strength of concrete
fy = specified yield strength of tensile steel reinforcement
h = height of the compression face of the member
Jd = distance between the resultants of the internal compressive and tensile forces on a cross section
L = length of span
Mn = nominal moment strength
Mu = moment due to factored load
Φ = strength reduction factor
w = uniformly distributed load

Glossary

Compression – the result of the subjection of a material to negative stress
Flexure – bending caused by an applied moment
Moment – a twisting force
Strain – relative displacement and deformation of a member
Stress – force over an area
Tension – the magnitude of the pulling stress
Yield strength – the stress at which a material begins to plastically deform