Introduction

- In prestressed concrete, a prestress force is applied to a concrete member and this induces an axial compression that counteracts all, or part of, the tensile stresses set up in the member by applied loading.

- In the field of bridge engineering, the introduction of prestressed concrete has aided the construction of long-span concrete bridges. These often comprise precast units, lifted into position and then tensioned against the units already in place, the process being continued until the span is complete.

- For smaller bridges, the use of simply supported precast prestressed concrete beams has proved an economical form of construction.

- The introduction of ranges of standard beam section has simplified the design and construction of these bridges.
Methods of Prestressing

- **Pre-tensioning** is used to describe a method of prestressing in which the tendons are tensioned before the concrete is placed, and the prestress is transferred to the concrete when a suitable cube strength is reached.

- **Post-tensioning** is a method of prestressing in which the tendon is tensioned after the concrete has reached a suitable strength. The tendons are anchored against the hardened concrete immediately after prestressing.
# Pre-tensioning Method

<table>
<thead>
<tr>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
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<tbody>
<tr>
<td>Tendons and reinforcement are positioned in the beam mould.</td>
<td>Tendons are stressed to about 70% of their ultimate strength.</td>
<td>Concrete is cast into the beam mould and allowed to cure to the required initial strength.</td>
<td>When the concrete has cured the stressing force is released and the tendons anchor themselves in the concrete.</td>
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</table>
# Post-tensioning Method

<table>
<thead>
<tr>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cable ducts and reinforcement are positioned in the beam mould. The ducts are usually raised towards the neutral axis at the ends to reduce the eccentricity of the stressing force.</td>
<td>Concrete is cast into the beam mould and allowed to cure to the required initial strength.</td>
<td>Tendons are threaded through the cable ducts and tensioned to about 70% of their ultimate strength.</td>
<td>Wedges are inserted into the end anchorages and the tensioning force on the tendons is released. Grout is then pumped into the ducts to protect the tendons.</td>
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</table>
In contrast to reinforced concrete, the design of prestressed concrete members is initially based upon the flexural behaviour at working load conditions.

The ultimate strength of all members in bending, shear and torsion is then checked, after the limit states of serviceability have been satisfied.

The prime function of prestressing is to ensure that only limited tensile stresses occur in the concrete under all conditions within the working range of loads.

To satisfy the limit state of cracking it is necessary to satisfy the stress limitations for the outermost fibres of a section.
Design for Class 1&2

- Imposed load
- Estimate self weight
- Dead load
- Bending moment and shear force diagrams
- Choose concrete grade and allowable stresses
- Determine min. $Z_1$ and $Z_2$
- Choose section
- Choose min. cover and determine max. $e$
- Estimate prestress losses
- Draw Magnet diagram
- Choose no. and sizes of tendons
- Determine cable zone and choose cable profile
- Determine prestress losses
- Determine ultimate flexural strength
- Determine ultimate shear strength
- End-block design or transmission length
- Determine deflections
- Detailing
Stress Limitation BS8110

- In general the stress limitations adopted for bridges are identical to BS8110 : Part 1: Clause 4.1.3. When considering the serviceability limit state of cracking of prestressed concrete members, three classifications of structural members are given:
  - Class 1: No tensile stresses;
  - Class 2: Flexural tensile stresses, but no visible cracking;
  - Class 3: Flexural tensile stresses, but surface crack widths not exceeding a maximum value (0.1mm for members in aggressive environments and 0.2mm for all other members)
Class of PSC Structure
## Limiting Stresses

The allowable compressive and tensile stresses for bonded Class 1 and Class 2 members at transfer and service load are provided by BS8110 and summarised as follows:

<table>
<thead>
<tr>
<th></th>
<th>Transfer Condition</th>
<th>Service Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compression</strong></td>
<td>0.50 $f_{ci}$</td>
<td>0.33$f_{cu}$</td>
</tr>
<tr>
<td><strong>Tension</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class 1</td>
<td>1.0 N/mm²</td>
<td>0</td>
</tr>
<tr>
<td>Class 2: Pretensioned</td>
<td>0.45 $\sqrt{f_{ci}}$</td>
<td>0.45 $\sqrt{f_{cu}}$</td>
</tr>
<tr>
<td>Postensioned</td>
<td>0.36 $\sqrt{f_{ci}}$</td>
<td>0.36 $\sqrt{f_{cu}}$</td>
</tr>
</tbody>
</table>
Basic Theory
Basic Inequalities

Stresses at transfer condition

Top fibre
\[ \frac{\alpha P_i}{A_c} - \frac{\alpha P_e}{Z_t} + \frac{M_i}{Z_t} \geq f'_{\min} \]

Bottom fibre
\[ \frac{\alpha P_i}{A_c} + \frac{\alpha P_e}{Z_b} - \frac{M_i}{Z_b} \leq f'_{\max} \]
Basic Inequalities

Stresses at service condition

Top fibre \( \frac{\beta P_i}{A_c} - \frac{\beta P_e}{Z_t} + \frac{M_s}{Z_t} \leq f_{\text{max}} \)

Bottom fibre \( \frac{\beta P_i}{A_c} + \frac{\beta P_e}{Z_b} - \frac{M_s}{Z_b} \geq f_{\text{min}} \)
Inequalities for $Z_t$ and $Z_b$

- Re-arranging the above inequalities by combining, the expressions for $Z_t$ and $Z_b$ can be obtained.

- These two inequalities may be used to estimate the preliminary section for design.

\[
Z_t \geq \frac{\left( \alpha M_s - \beta M_i \right)}{\left( \alpha f_{\max} - \beta f_{\min}' \right)}
\]

\[
Z_b \leq \frac{\left( \alpha M_s - \beta M_i \right)}{\left( \beta f_{\max}' - \alpha f_{\min} \right)}
\]
Inequalities for Prestress Force $P$

\[
P_i \geq \frac{(Z_t f'_{\text{min}} - M_i)}{\alpha(Z_t / A_c - e)}
\]

\[
P_i \leq \frac{(Z_b f'_{\text{max}} + M_i)}{\alpha(Z_b / A_c + e)}
\]

\[
P_i \leq \frac{(Z_t f_{\text{max}} - M_s)}{\beta(Z_t / A_c - e)}
\]

\[
P_i \geq \frac{(Z_b f_{\text{min}} + M_s)}{\beta(Z_b / A_c + e)}
\]
Prestress Losses

- Elastic deformation of concrete (Clause 4.8.3 BS8110)
- Anchorage draw-in (Clause 4.8.6)
- Friction losses (Clause 4.9 BS8110)
- Concrete shrinkage (Clause 4.8.4)
- Concrete creep (Clause 4.8.5)
- Steel relaxation (Clause 4.8.2)
Elastic deformation of concrete
(Clause 4.8.3 BS8110)

- As the concrete is compressed an elastic shortening of the member occurs. This movement is accompanied by an equal reduction in length of the prestressing steel resulting in loss in prestress force.
- For pretensioned beam, loss = \(mf_{co}\) where

\[
f_{co} = \frac{f_{pi}}{m + \frac{A_c}{A_{ps}(1 + e^2/r^2)}}
\]

- If the tendons are closely grouped in the tensile zone, the loss due to elastic shortening may be found by taking \(f_{co}\) as the stress in concrete at the level of the centroid of the tendons.
Elastic deformation of concrete  
(Clause 4.8.3 BS8110)

- For post-tensioned beam, loss = \( \frac{1}{2} m f_{co} \) where,

\[
f_{co} = \frac{f_{pi}}{m + \frac{A_c}{A_{ps} (1 + w^2/r^2)}} - \frac{M_i e}{I}
\]

- The value of \( f_{co} \) will vary along the member, since generally both \( e \) and \( M_i \) will vary. In this case an average value of \( f_{co} \) should be assumed.
Anchorage draw-in (Clause 4.8.6)

- When cables are anchored in a post-tensioned member, there is a ‘draw-in’ at the wedges, which may amount to 5-10mm for each cable depending on the system used.
- Loss in prestress force = \((s/L)(E_s)A_{ps}\) kN
Friction losses (Clause 4.9  BS8110)

In post-tensioned concrete there are four causes of friction loss to be considered:

1) Between the cable and the end-anchorage.
2) Developed inside the jack as the cable passes through it.
3) Caused by the unintentional variation in the duct alignment known as ‘wobble’ of the duct. This loss is described by a ‘wobble factor’ $K$ which varies with the rigidity of the duct, the frequency and the strength of the duct supports. (Equation 58, Clause 4.9.3, BS8110)
4) Due to curvature of the cable duct and the co-efficient of friction $\mu$ between the cable and the duct. (Equation 59, Clause 4.9.4, BS8110)
Friction Losses

Friction due to ‘wobble’
(Equation 58, Clause 4.9.3, BS8110)

Friction due to curvature
(Equation 59, Clause 4.9.4, BS8110)
Friction losses
(Clause 4.9  BS8110)

- The combined effect of curvature and wobble gives the variation in prestress force at a distance \( x \) from the jack as follows:

\[
P_x = P_0 e^{-(\mu \theta + Kx)}
\]

- For a parabolic cable profile since \( \theta \) represents the change in slope it is a linear function of \( x \). Thus prestress force \( P_o \) from the jack decreases linearly with distance. For a circular arc, \( \theta = L/R \) is the change in slope or ‘angle consumed’
Concrete shrinkage (Clause 4.8.4)

- The shrinkage strain $\varepsilon_{sh}$ is taken as $300 \times 10^{-6}$ for pre-tensioned work and $200 \times 10^{-6}$ for post-tensioned concrete where stressing is assumed to take place 2 – 3 weeks after concreting.

- Normally, half the total shrinkage takes place in the first month after transfer and $\frac{3}{4}$ of the total in the first 6 months.

- Loss in prestress force = $\varepsilon_{sh} (E_s) A_{ps}$ kN
Concrete creep (Clause 4.8.5)

- The creep strain used for calculating creep loss is given by: \( \varepsilon_r = \frac{\phi f_c}{E_{ci}} \)
- \( \phi = \) creep coefficient = 1.8 for transfer at 3 to 7 days or 1.4 for transfer after 28 days.
- Loss of prestress = \( \varepsilon_r (E_s) A_{ps} \) kN
Steel relaxation (Clause 4.8.2)

- The long-term relaxation loss is specified in BS8110 as the 1000-hour relaxation test value given by the tendon manufacturer.

Loss of Prestress =

\[
\text{Relaxation factors} \times \text{1000hour test value} \]

Table 4.6 (BS8110) \hspace{1cm} \text{(Clause 4.8.2.2 BS8110)}

- The creep loss may be assumed to take place at the same time and in the same manner as the shrinkage loss.
Total Prestress Losses

- If the initial prestress force applied to a member is $P_i$, then the effective prestress force at transfers is $\alpha P_i$, while that at service load is $\beta P_i$.
- The value of $\alpha$ reflects the short-term losses due to elastic shortening, anchorage draw-in and friction.
- Total loss coefficient $\beta$ accounts for the short term and long-term time-dependent losses due to concrete shrinkage and creep and steel relaxation.
Magnel Diagram

- The relationship between $1/P_i$ and $e$ are linear and if plotted graphically, they provide a useful means of determining appropriate values of $P_i$ and $e$.

- These diagrams were first introduced by a Belgian engineer, Magnel and hence the name Magnel Diagram.

\[
\begin{align*}
\frac{1}{P_i} &\leq \frac{\alpha(Z_t/A_c - e)}{(Z_tf_{\min}' - M_i)} \\
\frac{1}{P_i} &\geq \frac{\alpha(Z_b/A_c + e)}{(Z_bf_{\max}' + M_i)} \\
\frac{1}{P_i} &\geq \frac{\beta(Z_t/A_v - e)}{(Z_tf_{\max}' - M_s)} \\
\frac{1}{P_i} &\leq \frac{\beta(Z_b/A_c + e)}{(Z_bf_{\min}' + M_s)}
\end{align*}
\]
Magnel Diagram

\[ \frac{10^8}{P_i} \]

e

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Cable Zone and Cable Profile

- Once the prestress force has been chosen based on the most critical section, it is possible to find the limits of the eccentricity $e$ at sections elsewhere along the member.
- An allowable cable zone is produced within which the profile may take any shape.
- The term ‘cable’ is used to denote the resultant of all the individual tendons.
- As long as the ‘cable’ lies within the zone, the stresses at the different loading stages will not exceed the allowable values, even though some of the tendons might physically lie outside the cable zone.
Cable Zone and Cable Profile

These inequalities may be used to plot the permissible cable zone along the beam and help to determine the profile of the tendons.

\[
e \leq \frac{Z_t}{A_c} + \frac{1}{\alpha P_i} \left( M_i - Z_t f_{\min}' \right)
\]

\[
e \leq \frac{1}{\alpha P_i} \left( Z_b f_{\max}' + M_i \right) - \frac{Z_b}{A_c}
\]

\[
e \geq \frac{Z_t}{A_c} + \frac{1}{\beta P_i} \left( M_s - Z_t f_{\max}' \right)
\]

\[
e \geq \frac{1}{\beta P_i} \left( Z_b f_{\min}' + M_s \right) - \frac{Z_b}{A_c}
\]
Shear in Prestressed Concrete Beam

- The shear resistance of prestressed concrete members at the ultimate limit state is dependent on whether or not the section in the region of greatest shear force has cracked.
- The mode of failure is different for the two cases. If the section is uncracked in flexure, then failure in shear is initiated by cracks which form in the webs of I or T sections once the principal tensile strength has been exceeded.
- If the section is cracked, then failure is initiated by cracks on the tension face of the member extending into the compression zone, in a similar manner to the shear mode for reinforced concrete members.
Shear in Prestressed Beam

Cracks in tension face

Cracks extending into compression zone
Shear resistance of uncracked sections, $V_{\text{co}}$

- Equation 54 in BS8110

$$V_{\text{co}} = 0.67b_v h \sqrt{\left( f_{\text{prt}}^2 + 0.8 f_{\text{prt}} f_{\text{cp}} \right)}$$

- The values of $V_{\text{co}}/bh$ for different concrete grades and levels of prestress are given in Table 4.5 in BS8110.

- For I and T-sections, the maximum principal tensile stress occurs at the junction of the flange and the web (where the value of $A_y > 0.67bh$) and the equation gives a reasonable approximation to the uncracked shear resistance if the centroid lies within the web.

- If the centroid lies within the flange, the principal tensile stress at the junction should be limited to $0.24 \sqrt{f_{\text{cu}}}$ and $f_{\text{cp}}$ should be 0.8 of the prestress in the concrete at the junction.
Shear resistance of cracked sections, $V_{cr}$

- Equation 55 in BS8110 gives an empirical expression for the ultimate cracked shear resistance of beams

$$V_{cr} = \left( 1 - 0.55 \frac{f_{pe}}{f_{pu}} \right) v_c b d + M_o \frac{V}{M}$$

where

$$V_{cr} \geq 0.1 b_v d \sqrt{f_{cu}}$$

$$f_{pe} = \frac{\beta P_i}{A_{ps}}$$
Shear resistance of uncracked sections, $V_{co}$

- BS8110 Clause 4.3.8.2 requires that the maximum shear stress at any section should under no circumstances exceed $0.8\sqrt{f_{cu}}$ or 5 N/mm$^2$ whichever is less whether the section is cracked or uncracked.

- In determining this maximum stress, the reduction in web width due to un-grouted post-tensioned ducts should be considered. Even for grouted construction, only the concrete plus one third of the duct width should be used in finding the maximum shear stress.

- The lateral spacing of the legs of the links across a section should not exceed $d$. (See Clause 4.3.8.9 and 4.3.8.10)
Shear reinforcement

- If the shear resistance of a prestressed concrete member is not sufficient, then shear reinforcement must be provided in the form of links, similar to those used in reinforced concrete members.
- BS8110 states that, shear reinforcement is not required in cases where the ultimate shear force at a section is less than $0.5V_c$, where $V_c$ is based on the lesser of $V_{co}$ and $V_{cr}$, or when the member is of minor importance.
- Where minimum links are provided in a member, the shear resistance of these links is added to that of the member, $V_c$. The cross-sectional area $A_{sv}$, of the minimum links at a section are given by,

$$A_{sv}/s_v = 0.4b/(0.87f_{yv})$$

Where $b =$ breadth of the member (or of the rib in I or T section) $s_v =$ spacing of links along the member $f_{yv} =$ characteristic strength of shear reinforcement
Shear reinforcement

- The total shear resistance, $V_r$ of a member with nominal reinforcement is given by,
  $$V_r = V_c + 0.4d$$
  Where $d$ = depth to the centroid of the tendons and longitudinal reinforcement

- If the shear force at a given section exceeds the value given by $V_r$ in above equation, then the total shear force in excess of $V_c$ must be resisted by the shear reinforcement, and the amount is then given by,
  $$A_{sv}/s_v = (V-V_c)/(0.87f_y d)$$

- See Clauses 4.3.8.6, 4.3.8.7 and 4.3.8.8 and Table 3.8 of BS8110.
Ultimate Strength of Prestressed Concrete

- After designing a member to meet the stress limitations for serviceability, it is necessary to check the ultimate limit state.
- The section analysis is carried out by the method of strain compatibility in a similar manner to reinforced concrete members.
- The prestressing steel has an initial pre-strain which must be included in the strains derived from the strain diagram in flexure.
- The analysis of sections in Class 1 and 2 members at the serviceability limit state is carried out by treating the section as linearly elastic and using ordinary bending theory.
Ultimate Strength of Prestressed Concrete

- Class 1 and 2 members are assumed to remain uncracked at the service loads, justifying the use of a value of second moment of area based on the gross-concrete section.
- For Class 3 members, however, the concrete is assumed to have cracked and the aim is to limit the crack widths to acceptable levels depending on the degree of exposure of the member.
- The stress-strain curves for steel and concrete and the simplified concrete stress block are given in the BS8110 to enable ultimate-strength calculations to be carried out quickly by hand.
- Code formula (BS8110) and design charts (CP110:1972: Part3) are also available for analyzing rectangular beams and for T-beams where the neutral axis lies within the compression flange.
Simplified Stress-Strain Curve
Simplified Stress Strain BS8110

$\varepsilon_{pe}$  $\varepsilon_p$  $0.0035$  $0.45f_{cu}$

$x$  $0.9x$  $z$

$C$  $T$
Ultimate Strength of Prestressed Concrete

- For members with unbonded tendons, the effect of unbonding at the serviceability limit state is very small, but the behaviour at the ultimate limit state is markedly different.
- The ultimate moment of resistance of an unbonded section is generally smaller than that for a similar bonded section.
- The analysis of unbonded sections at the ultimate limit state cannot be carried out based on the basic principles since the strain is no longer equal to the strain in the concrete at the same level because there is no bond between the two materials. (Grouting of post-tensioned tendons after tensioning is sometime not done due to cost and time-consuming).
- BS8110 formula may be used for unbonded sections with different values for $f_{pb}$ and $x$. 
Ultimate Strength of Prestressed Concrete

- Equation 51 in BS8110 provides a formula for calculating ultimate moment of resistance for rectangular section or flanged section where the neutral axis lies in the flange. (Check $C>T$ and $h_f > 0.9x$).

- The values of $f_{pb}$ and $x$ for such sections may be obtained from Table 4.4 of Clause 4.3.7.3 in BS8110.

\[ Mu = f_{pb}A_{ps}(d-0.45x) \text{ where } d_n = 0.45x \]

Where $f_{pb} = \text{tensile stress in the tendon during failure}$
$A_{ps} = \text{area of tendons in tension zone}$
$d = \text{effective depth to centroid for } A_{ps}$
$x = \text{depth of neutral axis}$
Ultimate Strength of Prestressed Concrete

For unbonded tendons, the values of $f_{pb}$ and $x$ may be obtained from Equation 52 and Equation 53 in BS8110 as follows:

**Equation 52**

$$f_{pb} = f_{pe} + \frac{7000}{(l/d)} \left[ 1 - 1.7 \frac{f_{pu}A_{ps}}{f_{cu}bd} \right] \leq 0.7 f_{pu}$$

**Equation 53**

$$x = 2.47 \left[ \frac{f_{pu}A_{ps}}{f_{cu}bd} \right] \left[ \frac{f_{pb}}{f_{pu}} \right] d$$

- $f_{pe} = \text{effective design prestress in tendon after all losses}$
- $f_{pu} = \text{characteristic strength of tendons}$
- $l = \text{distance between two anchorages}$
- $b = \text{width of rectangular beam or effective width of flanged beam}$
Deflection of Prestressed Beams

- The deflection of prestressed beams is difficult to assess in practice since it is dependent upon many variables as follows:
  - Elastic deflection due to prestress
  - Elastic deflection due to initial loading
  - Creep deflection under sustained stresses
  - Deflection due to loss of prestress
  - Additional deflection due to live load

- The deflection due to prestress may be calculated by treating the prestress as an equivalent normal loading. Since concrete deforms both instantaneously under load and also with time, due to creep, the deflections of concrete structures should be assessed under both short-term and long-term conditions.
Deflection of Prestressed Beams

- Prestressed concrete members differ from reinforced concrete ones with regard to deflections as follows:

  (1) Deflections under a given load can be eliminated entirely by the use of a suitable arrangement of prestressing.
  (2) Deflections in PSC members usually occur even with no applied load (this is known as camber) and is generally an upwards deflection.

- The use of prestress to control deflections makes it difficult to specify span/depth ratios for initial estimation of member size. General rough guidelines may be given for simply supported beams: For bridge beams carrying heavy loads, a span/depth ratio in the range of 20-26 for Class 1 members, while for Class 2 or 3 floor or roof beams, a range of span/depth ratios is 26-30.
Short-term deflections for Class 1 and 2 members

- In order to determine the deflections of simply supported members under prestress force only, use is made of the fact that the moment in the member at any section $x$ is equal to $Pe(x)$ where $e(x)$ is the eccentricity at that section.

- The prestress moment diagram is thus proportional to the area between the member centroid and the location of the resultant prestressing force, as shown.
Short-term deflections for Class 1 and 2 members

- A simplified method of finding the maximum deflection of concrete members is outlined in BS8110 and is suitable for Class 3 members with low percentages of prestressing steel.
- In this case, the maximum deflection $y_{\text{max}}$ is given by $y_{\text{max}} = KL^2/r_b$ where $L$ is the effective span, $1/r_b$ is the curvature at midspan or at the support for a cantilever and $K$ is a constant which depends on the shape of the bending moment diagram.
Short-term deflections for Class 1 and 2 members

An alternative method of determining deflections is given by the code ACI318-77 which uses an effective second moment of area:

\[ I_e = \left( \frac{M_{cr}}{M_{max}} \right)^3 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_{max}} \right)^3 \right] I_{cr} \]

where \( I_g \) and \( I_{cr} \) are second moments of area of the gross and cracked sections respectively, \( M_{cr} \) is the bending moment to cause cracking at the tension face and \( M_{max} \) is the maximum bending moment in the member.
Long-term deflections

- Long term shrinkage and creep movements will cause the deflections of prestressed concrete members to increase with time.
- The effects of creep may be estimated by using a method given in BS8110 whereby an effective modulus of elasticity $E_{\text{ceff}}$ is given by $E_{\text{ct}} = E_{c28}/(1+\phi)$ where $E_{\text{ct}}$ is the instantaneous modulus of elasticity at the age considered and $\phi$ is the creep coefficient.
- The value of $E_{\text{ct}}$ may be estimated from BS8110 Part 2, Clause 7.2
Long-term deflections

Where only a proportion of the service load is permanent, the long-term curvature of a section may be found using the following procedure:

a) Determine the short-term curvature under the permanent load.
b) Determine the short-term curvature under the total load.
c) Determine the long-term curvature under the permanent load.

Total long-term curvature = curvature (c) + curvature (b) – curvature (a).
End Block Design

- There are 2 problems associated with end-block design namely, the assessment of the bursting tensile stresses and the compressive bearing stresses directly beneath the bearing plate.

- For post-tensioned members, the prestressing force in a tendon is applied through the anchorages as a concentrated force.

- By St. Venant’s principle, the stress distribution in a member is reasonably uniform away from the anchorage but in the region of the anchorage itself the stress distribution is complex.
End Block Design

- The most significant effect for design is that tensile stresses are set up transverse to the axis of the member, tending to split the concrete apart and reinforcement must be provided to contain the tensile stresses.

- The compressive bearing stress is controlled by the design of the anchors and the spacing between anchorages.

- A small helix if often welded to the anchor and provide additional precaution against poor compaction. It should not be considered as part of the reinforcement resisting tensile bursting stresses.
Bursting forces in anchorage zones

- The end-block of a concentrically-loaded post-tensioned member of rectangular cross-section and the distributions of principal tensile and compressive stresses within the end block is shown in the diagram below.
Bursting forces in anchorage zones

- The actual distribution of the bursting stresses is not uniform and complex but can be approximated to vary as shown in the diagram. The distribution can be further approximated by a triangle. It is sufficiently accurate to consider the resultant of these stresses, $F_{bst}$.

- At the ultimate limit state, $F_{bst}$ is assumed to act in a region extending from $0.2y_o$ to $2y_o$, where $y_o$ is half the side of the end-block. The value of $F_{bst}$ as a proportion of $P_i$ (initial jacking force) may be found from Table 4.7 BS8110. $F_{bst}$ depends on the ratio of $y_{po}/y_o$ where $y_{po}$ is half the side of the loaded area.
Approximation for Bursting Forces
Bursting forces in anchorage zones

- Circular loaded areas should be considered as square areas of equivalent cross-sectional area.

- For post-tensioned members with bonded tendons (i.e. grouted after tensioning) the bursting force $F_{bst}$ will be distributed in a region extending from $0.2y_o$ to $2y_o$ from the loaded face and should be resisted by reinforcement in the form of spirals or closed links, uniformly distributed throughout this region, and acting at a stress of 200 N/mm$^2$. 
Bursting forces in anchorage zones

- For members with unbonded tendons $F_{bst}$ should be assessed from Table 4.7 on the basis of the characteristic tendon force; the reinforcement provided to sustain this force may be assumed to be acting at its design strength $0.87f_y$.

- Where an end-block contains multiple anchorages, it should be divided into a series of symmetrically loaded prisms and then each prism treated as a separate end-block. Additional reinforcement should be provided around the whole group of anchorages to maintain overall equilibrium.
Transmission lengths in pretensioned members

- Once the tendons in a pretensioned member has been cut, the force in them which was initially maintained by the anchorages at the end of the pretensioning bed, is transferred suddenly to the ends of the concrete member. However, since there is no anchorage at the end of the member, as in the case of post-tensioning, there can be no force in the tendon there.

- Further along the tendon, the bond between the steel and the concrete enables the force in the tendons to build up, until some distance from the end of the member a point is reached where the force in the tendons equals the initial prestress force.

- This distance is known as transmission length (Clause 4.10 BS8110) which depends on degree of compaction of concrete; size and type of tendon; strength of the concrete; deformation and surface condition of the tendon. See Clause 4.10.3 in BS8110 for the calculation of the transmission length.
Composite Construction

- Many applications of prestressed concrete involve the combination of precast prestressed concrete beams and in-situ reinforced concrete slab.
- A common example is the in-situ infill between precast bridge beams. The beams are designed to act alone under their own weights plus the weight of the wet concrete of the slab. Once the concrete in the slab has hardened, provided that there is adequate horizontal shear connection between the slab and beam, they behave as a composite section under service load. The beam acts as permanent formwork for the slab, which provides the compression flange of the composite section.
- The section size of the beam can thus be kept to a minimum, since a compression flange is only required at the soffit at transfer. This leads to the use of inverted T sections.
Stress distribution within a composite section
Composite Construction

- The stress distribution is due to self weight of the beam with the maximum compressive stress at the lower extreme fiber.

- Once the slab is in place, the stress distribution in the beam is modified to take account the moment due combined section self-weight of the beam and slab, \( M_d \).

- Once the concrete in the slab has hardened and the imposed load acts on the composite section, the additional stress distribution is determined by using ordinary bending theory but using the composite section properties.

- The final stress distribution is a superposition of the modified stress distribution in beam and the combined section. There is a discontinuity in the final stress distribution under service load at the junction between the beam and slab.

- The beam has an initial stress distribution before it behaves as part of the composite section, whereas the slab only has stresses induced in it due to the composite action.
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- A post-tensioned prestressed concrete bridge deck is in the form of a solid slab and is simply supported over a span of 20m. It carries a service load of 10.3 kN/m². Assume Class 1 member with $f_{cu} = 50.6\text{N/mm}^2$, $f_{ci} = 40\text{N/mm}^2$ and the short-term and long-term losses to be 10% and 20% respectively.
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- Determine the allowable concrete stresses for the solid slab deck.
- Determine the minimum depth of slab required.
- If the depth of slab is 525mm and the maximum eccentricity of the tendons at midspan is 75mm above the soffit, find minimum value of the prestress force required.
- Construct a Magnel Diagram for the bridge slab and find the minimum prestress force for a tendon eccentricity of 188mm.
- Determine the cable zone for the full length of the bridge deck and a suitable cable profile.
- Determine the ultimate moment of resistance of the section at midspan with e=188mm. Assume $f_{pu}= 1770 \text{ N/mm}^2$, $f_{pi}=1239 \text{ N/mm}^2$, $f_{cu}=40 \text{ N/mm}^2$. $E_s = 195 \text{ kN/m}^2$, $f_y = 460 \text{ N/mm}^2$ and total steel area per metre = 4449 mm$^2$. Assume grouted tendons after tensioning.
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- From BS8110, for Class 1 member, the allowable stresses for the deck are:
  \[
  f'_{\text{max}} = 0.5 f_{ci} = 20.0 \text{ N/mm}^2 \\
  f_{\text{max}} = 0.33 f_{cu} = 16.7 \text{ N/mm}^2 \\
  f'_{\text{min}} = -1.0 \text{ N/mm}^2 \\
  f_{\text{min}} = 0 \text{ N/mm}^2
  \]

- Moments
  \[
  M_i = 24h \times 20^2/8 \quad \text{where } h \text{ is the overall depth of the slab.} \\
  M_s = 1200h + (10.3 \times 20^2)/8 = (1200h + 515) \text{ kNm/m}
  \]
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

Using the inequalities for $Z_t$ and $Z_b$, we have

\[
Z_t \geq \frac{0.9(1200h + 515) - 0.8(1200h)}{[0.9(16.7) - 0.8(-1)]} \times 10^6
\]

\[
= (7.58h + 29.28) \times 10^6 \text{ mm}^3/\text{m}
\]

\[
Z_b \geq \frac{0.9(1200h + 515) - 0.8(1200h)}{[0.8(20.0) - 0.9(0)]} \times 10^6
\]

\[
= (7.50h + 28.97) \times 10^6 \text{ mm}^3/\text{m}
\]
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- For a rectangular section, \( Z_t = Z_b = 10^3 \left( \frac{h^2}{6} \right)10^6 \)
  \[ = 0.167h^2 \times 10^9 \text{ mm}^3/\text{m} \]

- Thus the two equations can be formed for \( h \) as follows:
  \[ 0.167h^2 \times 10^9 = 7.58h + 29.28 \times 10^6 \]
  \[ 0.167h^2 \times 10^9 = 7.50h + 28.97 \times 10^6 \]

- Solving these two equations gives values for \( h \) of 0.442m and 0.440m and hence the minimum depth of the slab must be 442mm.

- When estimating initial size of section using the inequalities, it is better to use much larger depth to ensure that ultimate limit state is satisfied. This will also ensure that the effects of misplaced tendons during construction will be minimized.
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- Finding the minimum value of prestress force.
- Assuming a depth of 525mm is used for the deck slab.

\[ Z_t = Z_b = 525^2 \times 10^3/6 = 45.94 \times 10^6 \text{ mm}^3/\text{m} \]
\[ A_c = 5.25 \times 10^5 \text{ mm}^2/\text{m} \]
\[ e = 525/2 - 75 = 188 \text{ mm} \]
\[ M_i = 1200 (0.525) = 630 \text{ kNm/m} \]
\[ M_s = 630 + 515 = 1145 \text{ kNm/m} \]

- Use the inequalities for \( P_i \) to find value of prestress for a given \( e \), we have 4 sets of values for \( P \) as follows:

\[ P_i \leq 7473.4 \text{kN/m} \]
\[ P_i \leq 6246.3 \text{kN/m} \]
\[ P_i \geq 4699.3 \text{kN/m} \]
\[ P_i \geq 5195.0 \text{kN/m} \]

Thus the minimum value for \( P_i \) which lies within these limits is 5195 kN/m.
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- Constructing the Magnel Diagram.

1. \( \frac{10^8}{P_i} \geq 0.133e - 11.65 \)
2. \( \frac{10^8}{P_i} \geq 0.058e + 5.08 \)
3. \( \frac{10^8}{P_i} \leq 0.212e - 18.53 \)
4. \( \frac{10^8}{P_i} \leq 0.070e + 6.11 \)

- The signs of first and third inequalities have reversed since their denominators are negative.
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- By plotting the above inequalities with $10^8/Pi$ against $e$ then each linear relationship will define a feasible region in which the combination of $P$ & $e$ may lie without exceeding the limiting stresses.

- For any given eccentricity, we can see which pair of inequalities will give the limits for $Pi$. Thus for $e = 188$mm, the range of allowable values for $Pi$ is given by inequalities (2) and (4), i.e.

\[
\begin{align*}
\text{From inequality (2)} & \quad Pi \leq 6246.3 \text{ kN/m} \\
\text{From inequality (4)} & \quad Pi \geq 5195.0 \text{ kN/m}
\end{align*}
\]
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- The limits for the cable zone are given by the relevant inequalities:

\[
\begin{align*}
e & \leq 97.3 + 2.139 \times 10^{-7} \text{ Mi} \\
e & \leq 109.0 + 2.139 \times 10^{-7} \text{ Mi} \\
e & \geq -97.1 + 2.406 \times 10^{-7} \text{ Ms} \\
e & \geq -87.5 + 2.406 \times 10^{-7} \text{ Ms}
\end{align*}
\]
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- The values of $M_i$, $M_s$ and $e$ are calculated using two inequalities (lower and upper limits) for half span and are summarized as follows:

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>0</th>
<th>2.5</th>
<th>5.0</th>
<th>7.5</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_i$ (kNm)</td>
<td>0</td>
<td>275.6</td>
<td>472.5</td>
<td>590.6</td>
<td>630</td>
</tr>
<tr>
<td>$M_s$ (kNm)</td>
<td>0</td>
<td>500.9</td>
<td>858.8</td>
<td>1073.4</td>
<td>1145</td>
</tr>
<tr>
<td>$e \geq$ (mm)</td>
<td>-88</td>
<td>33</td>
<td>119</td>
<td>171</td>
<td>188</td>
</tr>
<tr>
<td>$e \leq$ (mm)</td>
<td>97</td>
<td>156</td>
<td>198</td>
<td>224</td>
<td>232</td>
</tr>
</tbody>
</table>

- The width of the zone at midspan is $232 - 188 = 44$mm which is sufficient to allow for any accuracies in locating the tendon ducts. However, for the chosen prestress force of $5195.0$ kN/m, the limit for $e = 188$mm as the maximum practical eccentricity for the slab.

- Thus, if the tendons are nominally fixed with $e=188$mm, a small displacement upwards would bring the prestress force outside the cable zone. In order to overcome this, the spacing of the tendons is decreased slightly from 265mm to 250mm giving an increased prestress force of $5512.0$ kN/m.
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- The limits to the cable zone at midspan are now 172mm and 224mm. If the shape of the chosen cable profile is parabolic, then for the midspan eccentricity of 188mm, the shape of the profile is given by:
  \[ y = \frac{4 \times 0.188}{20^2} x (20-x) \]

- where \( y \) is a coordinate measured from the centroid of the section (below centroid is positive). The coordinates of the curve along the length of the deck can be found, and these are used to fix the tendon ducts in position during construction. These coordinates lie within the revised cable zone based on \( P_i = 5512.0 \text{ kN/m} \).
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- The stress-strain curve for the particular grade of steel used is as shown followed by the stress and strain distributions.

- The strain $\varepsilon_{pe}$ in the prestressing steel at the ultimate limit state due to prestress only is given $\varepsilon_{pe} = (0.8 \times 1239)/(195 \times 10^3) = 0.00508$
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- The total strain in the steel $\varepsilon_{pb} = \varepsilon_{pe} + \varepsilon_{p}$ and $\varepsilon_{p}$ is determined from the strain diagram as follows:
  
  \[ 0.0035/x = \varepsilon_{p} / (450 - x) \]

  \[ \therefore \varepsilon_{p} = (450 - x) \times (0.0035/x) \]

- The stress in the steel is found from the stress-strain curve and the forces in the concrete and steel, C and T respectively, are then determined (see table shown). The neutral axis may thus be taken with sufficient accuracy to be 336mm, showing that the steel has not yielded.
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- The ultimate moment of resistance, $\text{Mult} = 5440(450-0.45\times336)\times10^{-3}$
  $= 1625.5 \text{ kNm/m}$
- The ultimate applied uniform load $= 1.4 \times 12.6 + 1.6 \times 10.3 = 34.1 \text{ kN/m}^2$
- The maximum ultimate bending moment, $M_{\text{applied}} = 34.1 \times 20^2/8 = 1705 \text{ kNm/m}$

<table>
<thead>
<tr>
<th>$x$ (mm)</th>
<th>$\varepsilon_p$</th>
<th>$\varepsilon_{pb}$</th>
<th>$f_{pb}$ (N/mm$^2$)</th>
<th>$T$ (kN)</th>
<th>$C$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.00175</td>
<td>0.00683</td>
<td>1256</td>
<td>5588</td>
<td>4860</td>
</tr>
<tr>
<td>330</td>
<td>0.00127</td>
<td>0.00635</td>
<td>1233</td>
<td>5486</td>
<td>5346</td>
</tr>
<tr>
<td>336</td>
<td>0.00119</td>
<td>0.00627</td>
<td>1223</td>
<td>5440</td>
<td>5443</td>
</tr>
</tbody>
</table>
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- Since $\text{Mult} < \text{M}_{\text{applied}}$, extra untensioned reinforcement is required. The effective depth for this extra steel is $525-50 = 475$ mm.

- In order to estimate the required amount of untensioned reinforcement $A_s$, it may be assumed initially that both the prestressing steel and the untensioned reinforcement have not yielded.

- If the neutral axis is taken as $370$ mm, an equilibrium equation can be written to determine $A_s$.

\[
0.45 \times 40 \times 10^3 \times 0.9 \times 370 = \left\{\left[(450-370)/370\right] \times 0.0035 + 0.00508\right\} \times 195 \times 10^3 \times 4449
\]
\[+ \left[(475-370)/370\right] \times 0.0035 \times 200 \times 10^3 A_s\]

\[\therefore A_s = 4683 \text{ mm}^2/\text{m}\]

- This will be provided by T32 bars at 150mm centers ($A_s = 5360 \text{ mm}^2/\text{m}$).
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

- The required \( A_s = 4683 \text{ mm}^2/\text{m} \) will be provided by T32 bars at 150mm centers (\( A_s = 5360 \text{ mm}^2/\text{m} \)).
- It is now necessary to check that the ultimate moment of resistance is greater than the applied bending moment. This is achieved by using a trial-and-error procedure and values are summarized in the following table.

<table>
<thead>
<tr>
<th>( x ) (mm)</th>
<th>( \varepsilon_p )</th>
<th>( \varepsilon_{pb} )</th>
<th>( \varepsilon_{st} )</th>
<th>( f_{pb} ) (N/mm(^2))</th>
<th>( f_{st} ) (N/mm(^2))</th>
<th>C (kN)</th>
<th>T (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>370</td>
<td>0.00076</td>
<td>0.00584</td>
<td>0.00099</td>
<td>1139</td>
<td>198</td>
<td>6129</td>
<td>5994</td>
</tr>
<tr>
<td>373</td>
<td>0.00072</td>
<td>0.00580</td>
<td>0.00096</td>
<td>1131</td>
<td>192</td>
<td>6061</td>
<td>6043</td>
</tr>
</tbody>
</table>
Example on Post-Tensioned Concrete Slab Bridge (Ref: M. K. Hurst)

The strain in the un-tensioned reinforcement is given by $\varepsilon_{st} = (475-x)(0.0035/x)$ and the corresponding stress $f_{st}$ is found from the appropriate stress-strain curve.

The depth of the neutral axis is 373mm and the ultimate moment of resistance is given by,

$$M_{ult} = [4449 \times 1131(450-373) + 5360 \times 192(475-0.45\times373)] \times 10^{-6}$$

$$= 1735.8 \text{ kNm/m}$$

$M_{ult} > M_{applied}$ therefore the section is satisfactory.