Chapter 2

AC to DC Converters

(Rectifiers)
Outline

2.1 Single-phase controlled rectifier
2.2 Three-phase controlled rectifier
2.3 Effect of transformer leakage inductance on rectifier circuits
2.4 Capacitor-filtered uncontrolled rectifier
2.5 Harmonics and power factor of rectifier circuits
2.6 High power controlled rectifier
2.7 Inverter mode operation of rectifier circuit
2.8 Thyristor-DC motor system
2.9 Realization of phase-control in rectifier circuits
2.1 Single-phase controlled (controllable) rectifier

2.1.1 Single-phase half-wave controlled rectifier

2.1.2 Single-phase bridge fully-controlled rectifier

2.1.3 Single-phase full-wave controlled rectifier

2.1.4 Single-phase bridge half-controlled rectifier
2.1.1 Single-phase half-wave controlled rectifier

Resistive load

\[ U_d = \frac{1}{2\pi} \int_{\alpha}^{\pi} \sqrt{2} U_2 \sin \omega t d(\omega t) = \sqrt{2} U_2 \left(1 + \cos \alpha \right) = 0.45 U_2 \frac{1 + \cos \alpha}{2} \] (2-1)

- Half-wave, single-pulse
- Triggering delay angle, delay angle, firing angle
2.1.1 Single-phase half-wave controlled rectifier

Inductive (resistor-inductor) load
Basic thought process of time-domain analysis for power electronic circuits

The time-domain behavior of a power electronic circuit is actually the combination of consecutive transients of the different linear circuits when the power semiconductor devices are in different states.

\[ L \frac{di_d}{dt} + Ri_d = \sqrt{2}U_2 \sin \omega t \]  
\[ \omega t = \alpha, \quad i_d = 0 \]

\[ i_d = -\frac{\sqrt{2}U_2}{Z} \sin(\alpha - \varphi) e^{\frac{R}{\alpha L}(\alpha - \alpha)} + \frac{\sqrt{2}U_2}{Z} \sin(\omega t - \varphi) \]
Single-phase half-wave controlled rectifier with freewheeling diode

Inductive load (L is large enough)

\[ I_{dVT} = \frac{\pi - \alpha}{\pi} I_d \]  
\[ \eta_{\Delta f} = \frac{2\pi}{\pi + \alpha}\eta_f \]  
\[ I_{VT} = \sqrt{\frac{1}{2\pi} \int_{-\alpha}^{\pi} I_d^2 d(\omega t)} = \sqrt{\frac{\pi - \alpha}{2\pi}} I_d \]

- Maximum forward voltage, maximum reverse voltage
- Disadvantages:
  - Only single pulse in one line cycle
  - DC component in the transformer current
2.1.2 Single-phase bridge fully-controlled rectifier

Resistive load

- For thyristor: maximum forward voltage, maximum reverse voltage
- Advantages:
  - 2 pulses in one line cycle
  - No DC component in the transformer current
2.1.2 Single-phase bridge fully-controlled rectifier

Resistive load

- Average output (rectified) voltage

\[
U_d = \frac{1}{\pi} \int_0^\pi \sqrt{2} U_2 \sin \omega t d(\omega t) = \frac{2\sqrt{2} U_2}{\pi} \frac{1 + \cos \alpha}{2} = 0.9 U_2 \frac{1 + \cos \alpha}{2} \tag{2-9}
\]

- Average output current

\[
I_d = \frac{U_d}{R} = \frac{2\sqrt{2} U_2}{\pi R} \frac{1 + \cos \alpha}{2} = 0.9 \frac{U_2}{R} \frac{1 + \cos \alpha}{2} \tag{2-10}
\]

- For thyristor

\[
I_{dVT} = \frac{1}{2} I_d = 0.45 \frac{U_2}{R} \frac{1 + \cos \alpha}{2} \tag{2-11}
\]

\[
I_{VT} = \sqrt{\frac{1}{2\pi} \int_0^\pi \left(\frac{\sqrt{2} U_2}{R} \sin \omega t\right)^2 d(\omega t)} = \frac{U_2}{\sqrt{2} R} \sqrt{\frac{1}{2\pi} \sin 2\alpha + \frac{\pi - \alpha}{\pi}} \tag{2-12}
\]

- For transformer

\[
I = I_2 = \sqrt{\frac{1}{\pi} \int_0^\pi \left(\frac{\sqrt{2} U_2}{R} \sin \omega t\right)^2 d(\omega t)} = \frac{U_2}{R} \sqrt{\frac{1}{2\pi} \sin 2\alpha + \frac{\pi - \alpha}{\pi}} \tag{2-13}
\]
2.1.2 Single-phase bridge fully-controlled rectifier

Inductive load (L is large enough)

\[ U_d = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} \sqrt{2} U_2 \sin \omega t d(\omega t) = \frac{2\sqrt{2}}{\pi} U_2 \cos \alpha = 0.9 U_2 \cos \alpha \]  \hspace{1cm} (2-15)

- Commutation
- Thyristor voltages and currents
- Transformer current
**Electro-motive-force (EMF) load**

**With resistor**

![Diagram of EMF load with resistor](image)

- Discontinuous current $i_d$

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**Diagram Details**

- $i_d$: Discontinuous current
- $u_d$: Voltage
- $R$: Resistor
- $E$: EMF
- $\omega t$: Angular frequency
- $I_d$: Current
- $\alpha$, $\theta$, $\delta$: Phase angles

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Electro-motive-force (EMF) load

With resistor and inductor

- When L is large enough, the output voltage and current waveforms are the same as ordinary inductive load.
- When L is at a critical value

\[
\begin{align*}
L &= \frac{2\sqrt{2}U^2}{\pi \omega I_{d\min}} = 2.87 \times 10^{-3} \frac{U^2}{I_{d\min}} \quad (2-17)
\end{align*}
\]
2.1.3 Single-phase full-wave controlled rectifier

- Transformer with center tap
- Comparison with single-phase bridge fully-controlled rectifier
2.1.4 Single-phase bridge half-controlled rectifier

- Half-control
- Comparison with fully-controlled rectifier
- Additional freewheeling diode
Another single-phase bridge half-controlled rectifier

Comparison with previous circuit:
- No need for additional freewheeling diode
- Isolation is necessary between the drive circuits of the two thyristors
Summary of some important points in analysis

- When analyzing a thyristor circuit, start from a diode circuit with the same topology. The behavior of the diode circuit is exactly the same as the thyristor circuit when firing angle is 0.
- A power electronic circuit can be considered as different linear circuits when the power semiconductor devices are in different states. The time-domain behavior of the power electronic circuit is actually the combination of consecutive transients of the different linear circuits.
- Take different principle when dealing with different load
  - For resistive load: current waveform of a resistor is the same as the voltage waveform
  - For inductive load with a large inductor: the inductor current can be considered constant
2.2 Three-phase controlled (controllable) rectifier

2.2.1 Three-phase half-wave controlled rectifier
(the basic circuit among three-phase rectifiers)

2.2.2 Three-phase bridge fully-controlled rectifier
(the most widely used circuit among three-phase rectifiers)
2.2.1 Three-phase half-wave controlled rectifier

Resistive load, $\alpha = 0^\circ$

- Common-cathode connection
- Natural commutation point
Resistive load, $\alpha = 30^\circ$
Resistive load, $\alpha = 60^\circ$
Resistive load, quantitative analysis

- When $\alpha \leq 30^\circ$, load current $i_d$ is continuous.

  $$U_d = \frac{1}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} \sqrt{2} U_2 \sin(\omega t) \, d(\omega t) = \frac{3\sqrt{6}}{2\pi} U_2 \cos \alpha = 0.517 U_2 \cos \alpha \quad (2-18)$$

- When $\alpha > 30^\circ$, load current $i_d$ is discontinuous.

  $$U_d = \frac{1}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\pi} \sqrt{2} U_2 \sin(\omega t) \, d(\omega t) = \frac{3\sqrt{2}}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{6}} \cos \left( \frac{\pi}{6} + \alpha \right) \, d(\omega t) = 0.675 \left[ 1 + \cos \left( \frac{\pi}{6} + \alpha \right) \right] \quad (2-19)$$

- Average load current

  $$I_d = \frac{U_d}{R} \quad (2-20)$$

- Thyristor voltages
Inductive load, L is large enough

Load current $i_d$ is always continuous.

$$U_d = \frac{1}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} \sqrt{2} U_2 \sin \omega t \, dt = \frac{3\sqrt{6}}{2\pi} U_2 \cos \alpha = 1.17 U_2 \cos \alpha \quad (2-18)$$

Thyristor voltage and currents, transformer current

$$I_2 = I_{VT} = \frac{1}{\sqrt{3}} I_d = 0.577 I_d \quad (2-23) \quad I_{VT(AV)} = \frac{I_{VT}}{1.57} = 0.368 I_d \quad (2-24)$$

$$U_{FM} = U_{RM} = 2.45 U_2 \quad (2-25)$$
2.2.2 Three-phase bridge fully-controlled rectifier

- Common-cathode group and common-anode group of thyristors
- Numbering of the 6 thyristors indicates the trigger sequence.
Resistive load, $\alpha = 0^\circ$
Resistive load, $\alpha = 30^\circ$
Resistive load, $\alpha = 60^\circ$
Resistive load, $\alpha = 90^\circ$
Inductive load, $\alpha = 0^\circ$
Inductive load, $\alpha = 30^\circ$
Inductive load, $\alpha = 90^\circ$
Quantitative analysis

Average output voltage

\[
U_d = \frac{1}{\pi} \int_{\frac{\pi}{3} + \alpha}^{\frac{2\pi}{3} + \alpha} \sqrt{6} U_2 \sin \omega t d(\omega t) = 2.34 U_2 \cos \alpha
\]  

(2-26)

For resistive load, when \( \alpha > 60^\circ \), load current \( i_d \) is discontinuous.

\[
U_d = \frac{3}{\pi} \int_{\frac{\pi}{3} + \alpha}^{\pi} \sqrt{6} U_2 \sin \omega t d(\omega t) = 2.34 U_2 \left[1 + \cos \left(\frac{\pi}{3} + \alpha\right)\right]
\]  

(2-27)

Average output current (load current)

\[
I_d = \frac{U_d}{R}
\]  

(2-20)

Transformer current

\[
I_2 = \sqrt{\frac{1}{2\pi} \left(I_d^2 \times \frac{2}{3} \pi + (-I_d)^2 \times \frac{2}{3} \pi\right)} = \sqrt{\frac{2\pi}{3}} I_d = 0.816 I_d
\]  

(2-28)

Thyristor voltage and current

- Same as three-phase half-wave rectifier

EMF load, L is large enough

- All the same as inductive load except the calculation of average output current

\[
I_d = \frac{U_d - E}{R}
\]  

(2-29)
2.3 Effect of transformer leakage inductance on rectifier circuits

In practical, the transformer leakage inductance has to be taken into account.

Commutation between thyristors thus can not happen instantly, but with a commutation process.
Commutation process analysis

- Circulating current $i_k$ during commutation

\[
u_b - u_a = 2L_B \frac{di_a}{dt}
\]

\[
i_k: \quad 0 \longrightarrow I_d
\]

\[
i_a = I_d - i_k: \quad I_d \longrightarrow 0
\]

\[
i_b = i_k: \quad 0 \longrightarrow I_d
\]

- Commutation angle

- Output voltage during commutation

\[
u_d = u_a + L_B \frac{di_k}{dt} = u_b - L_B \frac{di_k}{dt} = \frac{u_a + u_b}{2} \quad (2-30)
\]
Quantitative calculation

- **Reduction of average output voltage due to the commutation process**

\[
\Delta U_d = \frac{1}{2\pi/3} \int_{\alpha+\frac{5\pi}{6}}^{\alpha+\frac{5\pi}{6}+\gamma} (u_b - u_d) d(\alpha t) = \frac{3}{2\pi} \int_{\alpha+\frac{5\pi}{6}}^{\alpha+\frac{5\pi}{6}+\gamma} [u_b - (u_b - L_B \frac{di_k}{dt})] d(\alpha t)
\]

\[
= \frac{3}{2\pi} \int_{\alpha+\frac{5\pi}{6}}^{\alpha+\frac{5\pi}{6}+\gamma} L_B \frac{di_k}{dt} d(\alpha t) = \frac{3}{2\pi} \int_0^{I_d} \alpha L_B d_i_k = \frac{3}{2\pi} X_B I_d
\]

(2-31)

- **Calculation of commutation angle**

\[
\cos \alpha - \cos(\alpha + \gamma) = \frac{2 X_B I_d}{\sqrt{6} U_2}
\]

- \( I_d \uparrow, \gamma \uparrow \)
- \( X_B \uparrow, \gamma \uparrow \)
- For \( \alpha \leq 90^\circ, \alpha \downarrow, \gamma \uparrow \)
Summary of the effect on rectifier circuits

<table>
<thead>
<tr>
<th>Circuits</th>
<th>Single-phase full wave</th>
<th>Single-phase bridge</th>
<th>Three-phase half-wave</th>
<th>Three-phase bridge</th>
<th>m-pulse rectifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta U_d$</td>
<td>$\frac{X_B}{\pi} I_d$</td>
<td>$\frac{2X_B}{\pi} I_d$</td>
<td>$\frac{3X_B}{2\pi} I_d$</td>
<td>$\frac{3X_B}{\pi} I_d$</td>
<td>$\frac{mX_B}{2\pi} I_d$ (①)</td>
</tr>
<tr>
<td>$\cos \alpha - \cos(\alpha + \gamma)$</td>
<td>$\frac{I_d X_B}{\sqrt{2} U_2}$</td>
<td>$\frac{2I_d X_B}{\sqrt{2} U_2}$</td>
<td>$\frac{2X_B I_d}{\sqrt{6} U_2}$</td>
<td>$\frac{2 X_B I_d}{\sqrt{6} U_2}$</td>
<td>$\frac{I_d X_B}{\sqrt{2} U_2 \sin \frac{\pi}{m}}$ (②)</td>
</tr>
</tbody>
</table>

**Conclusions**

- Commutation process actually provides additional working states of the circuit.
- di/dt of the thyristor current is reduced.
- The average output voltage is reduced.
- Positive du/dt
- Notching in the AC side voltage
2.4 Capacitor-filtered uncontrolled (uncontrollable) rectifier

- Emphasis of previous sections
  - Controlled rectifier, inductive load
- Uncontrolled rectifier: diodes instead of thyristors
- Wide applications of capacitor-filtered uncontrolled rectifier
  - AC-DC-AC frequency converter
  - Uninterruptible power supply
  - Switching power supply

2.4.1 Capacitor-filtered single-phase uncontrolled rectifier

2.4.2 Capacitor-filtered three-phase uncontrolled rectifier
2.4.1 Capacitor-filtered single-phase uncontrolled rectifier

Single-phase bridge, RC load
2.4.1 Capacitor-filtered single-phase uncontrolled rectifier

Single-phase bridge, $RLC$ load
2.4.2 Capacitor-filtered three-phase uncontrolled rectifier

Three-phase bridge, $RC$ load
2.4.2 Capacitor-filtered three-phase uncontrolled rectifier

Three-phase bridge, \( RC \) load

Waveform when \( \omega RC \leq 1.732 \)

\[ \omega RC = \sqrt{3} \]

\[ \omega RC < \sqrt{3} \]
2.4.2 Capacitor-filtered three-phase uncontrolled rectifier

Three-phase bridge, RLC load
2.5 Harmonics and power factor of rectifier circuits

- Originating of harmonics and power factor issues in rectifier circuits
  - Harmonics: working in switching states—nonlinear
  - Power factor: firing delay angle causes phase delay
- Harmful effects of harmonics and low power factor
- Standards to limit harmonics and power factor

2.5.1 Basic concepts of harmonics and reactive power

2.5.2 AC side harmonics and power factor of controlled rectifiers with inductive load

2.5.3 AC side harmonics and power factor of capacitor-filtered uncontrolled rectifiers

2.5.4 Harmonic analysis of output voltage and current
2.5.1 Basic concepts of harmonics and reactive power

- For pure sinusoidal waveform
  \[ u(t) = \sqrt{2}U \sin(\omega t + \varphi_u) \]  
  (2-54)

- For periodic non-sinusoidal waveform
  \[ u(\omega t) = a_o + \sum_{n=1}^{\infty} (a_n \cos n \omega t + b_n \sin n \omega t) \]  
  (2-55)

  or

  \[ u(\omega t) = a_o + \sum_{n=1}^{\infty} c_n \sin(n \omega t + \varphi_n) \]  
  (2-56)

where

\[ c_n = \sqrt{a_n^2 + b_n^2} \quad a_n = c_n \sin \varphi \]

\[ \varphi_n = \arctan(a_n/b_n) \quad b_n = c_n \cos \varphi \]

- Fundamental component
- Harmonic components (harmonics)
Harmonics-related specifications

Take current harmonics as examples

❖ Content of $n$th harmonics

$$HRI_n = \frac{I_n}{I_1} \times 100\% \quad (2-57)$$

$I_n$ is the effective (RMS) value of $n$th harmonics.
$I_1$ is the effective (RMS) value of fundamental component.

❖ Total harmonic distortion

$$THD_i = \frac{I_h}{I_1} \times 100\% \quad (2-58)$$

$I_h$ is the total effective (RMS) value of all the harmonic components.
### Definition of power and power factor

For sinusoidal circuits

- **Active power**
  \[
  P = \frac{1}{2\pi} \int_0^{2\pi} u_i d (\omega t) = UI \cos \phi \quad (2-59)
  \]

- **Reactive power**
  \[
  Q = UI \sin \phi \quad (2-61)
  \]

- **Apparent power**
  \[
  S = UI \quad (2-60)
  \]
  \[
  S^2 = P^2 + Q^2 \quad (2-63)
  \]

- **Power factor**
  \[
  \lambda = \frac{P}{S} \quad (2-62)
  \]
  \[
  \lambda = \cos \phi \quad (2-64)
  \]
Definition of power and power factor

For non-sinusoidal circuits

- **Active power**
  \[ P = UI_1 \cos \phi_1 \quad (2-65) \]

- **Power factor**
  \[ \lambda = \frac{P}{S} = \frac{UI_1 \cos \phi_1}{UI} = \frac{I_1}{I} \cos \phi_1 = \nu \cos \phi_1 \quad (2-66) \]

- **Distortion factor (fundamental-component factor)**
  \[ \nu = \frac{I_1}{I} \]

- **Displacement factor (power factor of fundamental component)**
  \[ \lambda_1 = \cos \phi_1 \]

- **Definition of reactive power is still in dispute.**
Review of the reactive power concept

- The reactive power $Q$ does not lead to net transmission of energy between the source and load. When $Q \neq 0$, the rms current and apparent power are greater than the minimum amount necessary to transmit the average power $P$.

- Inductor: current lags voltage by $90^\circ$, hence displacement factor is zero. The alternate storing and releasing of energy in an inductor leads to current flow and nonzero apparent power, but $P = 0$. Just as resistors consume real (average) power $P$, inductors can be viewed as consumers of reactive power $Q$.

- Capacitor: current leads voltage by $90^\circ$, hence displacement factor is zero. Capacitors supply reactive power $Q$. They are often placed in the utility power distribution system near inductive loads. If $Q$ supplied by capacitor is equal to $Q$ consumed by inductor, then the net current (flowing from the source into the capacitor-inductive-load combination) is in phase with the voltage, leading to unity power factor and minimum rms current magnitude.
2.5.2 AC side harmonics and power factor of controlled rectifiers with inductive load

Single-phase bridge fully-controlled rectifier
AC side current harmonics of single-phase bridge fully-controlled rectifier with inductive load

\[ i_2 = \frac{4}{\pi} I_d \left( \sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \cdots \right) \]

\[ = \frac{4}{\pi} I_d \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin n\omega t = \sum_{n=1,3,5,\ldots}^{\infty} \sqrt{2} I_n \sin n\omega t \]  \hspace{1cm} (2-72)

where

\[ I_n = \frac{2\sqrt{2} I_d}{n\pi} \]

\[ n=1,3,5,\ldots \]  \hspace{1cm} (2-73)

**Conclusions**

- Only odd order harmonics exist
- \( I_n \propto 1/n \)
- \( I_n / I_1 = 1/n \)
Power factor of single-phase bridge fully-controlled rectifier with inductive load

**Distortion factor**

\[ \nu = \frac{I_1}{I} = \frac{2\sqrt{2}}{\pi} \approx 0.9 \quad (2-75) \]

**Displacement factor**

\[ \lambda_1 = \cos \varphi_1 = \cos \alpha \quad (2-76) \]

**Power factor**

\[ \lambda = \nu \lambda_1 = \frac{I_1}{I} \cos \varphi_1 = \frac{2\sqrt{2}}{\pi} \cos \alpha \approx 0.9 \cos \alpha \quad (2-77) \]
Three-phase bridge fully-controlled rectifier
AC side current harmonics of three-phase bridge fully-controlled rectifier with inductive load

\[
i_a = \frac{2\sqrt{3}}{\pi} I_d \left[ \sin \omega t - \frac{1}{5} \sin 5 \omega t - \frac{1}{7} \sin 7 \omega t + \frac{1}{11} \sin 11 \omega t + \frac{1}{13} \sin 13 \omega t - \cdots \right] \\
= \frac{2\sqrt{3}}{\pi} I_d \sin \omega t + \frac{2\sqrt{3}}{\pi} I_d \sum_{n=6k \pm 1}^{k=1,2,3\cdots} \frac{(-1)^k}{n} \sin n \omega t = \sqrt{2} I_1 \sin \omega t + \sum_{n=6k \pm 1}^{k=1,2,3\cdots} (-1)^k \sqrt{2} I_n \sin n \omega t
\]

where

\[
\begin{align*}
I_1 & = \frac{\sqrt{6}}{\pi} I_d \\
I_n & = \frac{\sqrt{6}}{n \pi} I_d, \quad n = 6k \pm 1, \quad k = 1,2,3,\cdots
\end{align*}
\]

Conclusions

- Only 6k±1 order harmonics exist (k is positive integer)
- \( I_n \propto 1/n \)
- \( I_n / I_1 = 1/n \)
Power factor of three-phase bridge fully-controlled rectifier with inductive load

- **Distortion factor**

\[ \nu = \frac{I_1}{I} = \frac{3}{\pi} \approx 0.955 \]  \hspace{1cm} (2-81)

- **Displacement factor**

\[ \lambda_1 = \cos \varphi_1 = \cos \alpha \]  \hspace{1cm} (2-82)

- **Power factor**

\[ \lambda = \nu \lambda_1 = \frac{I_1}{I} \cos \varphi_1 = \frac{3}{\pi} \cos \alpha \approx 0.955 \cos \alpha \]  \hspace{1cm} (2-83)
2.5.3 AC side harmonics and power factor of capacitor-filtered uncontrolled rectifiers

- Situation is a little complicated than rectifiers with inductive load.
- Some conclusions that are easy to remember:
  - Only odd order harmonics exist in single-phase circuit, and only $6k \pm 1$ ($k$ is positive integer) order harmonics exist in three-phase circuit.
  - Magnitude of harmonics decreases as harmonic order increases.
  - Harmonics increases and power factor decreases as capacitor increases.
  - Harmonics decreases and power factor increases as inductor increases.
2.5.4 Harmonic analysis of output voltage and current

\[ u_{d0} = U_{d0} + \sum_{n=mk}^{\infty} b_n \cos n \omega t \]

\[ = U_{d0} \left[ 1 - \sum_{n=mk}^{\infty} \frac{2 \cos k \pi}{n^2 - 1} \cos n \omega t \right] \]

where

\[ U_{d0} = \sqrt{2} U_2 \frac{m}{\pi} \sin \frac{\pi}{m} \] \hspace{1cm} (2-86)

\[ b_n = -\frac{2 \cos k \pi}{n^2 - 1} U_{d0} \] \hspace{1cm} (2-87)

Output voltage of \( m \)-pulse rectifier when \( \alpha = 0^\circ \)
Ripple factor in the output voltage

**Output voltage ripple factor**

\[ \gamma_u = \frac{U_R}{U_{d0}} \]  
(2-88)

where \( U_R \) is the total RMS value of all the harmonic components in the output voltage

\[ U_R = \sqrt{\sum_{n=mk}^{\infty} U_n^2} = \sqrt{U^2 - U_{d0}^2} \]  
(2-89)

and \( U \) is the total RMS value of the output voltage

**Ripple factors for rectifiers with different number of pulses**

<table>
<thead>
<tr>
<th>( m )</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_u ) (%)</td>
<td>48.2</td>
<td>18.27</td>
<td>4.18</td>
<td>0.994</td>
<td>0</td>
</tr>
</tbody>
</table>
Harmonics in the output current

\[ i_d = I_d + \sum_{n=mk}^{\infty} d_n \cos(n\omega t - \varphi_n) \quad (2-92) \]

where

\[ I_d = \frac{U_{d0} - E}{R} \quad (2-93) \]

\[ d_n = \frac{b_n}{z_n} = \frac{b_n}{\sqrt{R^2 + (n\omega L)^2}} \quad (2-94) \]

\[ \varphi_n = \arctan \left( \frac{n\omega L}{R} \right) \quad (2-95) \]
Conclusions for $\alpha = 0^\circ$

- Only $mk$ ($k$ is positive integer) order harmonics exist in the output voltage and current of m-pulse rectifiers.

- Magnitude of harmonics decreases as harmonic order increases when $m$ is constant.

- The order number of the lowest harmonics increases as $m$ increases. The corresponding magnitude of the lowest harmonics decreases accordingly.
For $\alpha \neq 0^\circ$

- Quantitative harmonic analysis of output voltage and current is very complicated for $\alpha \neq 0^\circ$.

- As an example, for 3-phase bridge fully-controlled rectifier

$$u_d = U_d + \sum_{n=6k}^{\infty} c_n \cos (n \omega t + \theta_n)$$

(2-96)
2.6 High power controlled rectifier

2.6.1 Double-star controlled rectifier

2.6.2 Connection of multiple rectifiers
2.6.1 Double-star controlled rectifier

Circuit

Waveforms When $\alpha = 0^\circ$

Difference from 6-phase half-wave rectifier
Effect of interphase reactor (inductor, transformer)

\[ u_p = u_{d2} - u_{d1} \]

\[ u_d = u_{d2} - \frac{1}{2} u_p = u_{d1} + \frac{1}{2} U_p = \frac{1}{2} (u_{d1} + u_{d2}) \]
Quantitative analysis when \( \alpha = 0^\circ \)

\[
\begin{align*}
    u_{d1} &= \frac{3\sqrt{6}U_2}{2\pi} \left[ 1 + \frac{1}{4} \cos 3\omega t - \frac{2}{35} \cos 6\omega t + \frac{1}{40} \cos 9\omega t - \cdots \right] \quad (2-99) \\
    u_{d2} &= \frac{3\sqrt{6}U_2}{2\pi} \left[ 1 + \frac{1}{4} \cos 3(\omega t - 60^\circ) - \frac{2}{35} \cos 6(\omega t - 60^\circ) + \frac{1}{40} \cos 9(\omega t - 60^\circ) - \cdots \right] \\
    &= \frac{3\sqrt{6}U_2}{2\pi} \left[ 1 - \frac{1}{4} \cos 3\omega t - \frac{2}{35} \cos 6\omega t - \frac{1}{40} \cos 9\omega t - \cdots \right] \quad (2-100) \\
    u_p &= \frac{3\sqrt{6}U_2}{2\pi} \left[ -\frac{1}{2} \cos 3\omega t - \frac{1}{20} \cos 9\omega t - \cdots \right] \quad (2-101) \\
    u_d &= \frac{3\sqrt{6}U_2}{2\pi} \left[ 1 - \frac{2}{35} \cos 6\omega t - \cdots \right] \quad (2-102)
\end{align*}
\]
Waveforms when $\alpha > 0^\circ$

$U_d = 1.17U_2 \cos \alpha$
Comparison with 3-phase half-wave rectifier and 3-phase bridge rectifier

Voltage output capability
- Same as 3-phase half-wave rectifier
- Half of 3-phase bridge rectifier

Current output capability
- Twice of 3-phase half-wave rectifier
- Twice of 3-phase bridge rectifier

Applications
- Low voltage and high current situations
2.6.2 Connection of multiple rectifiers

Connection of multiple rectifiers

- To increase the output capacity
  - Larger output voltage: series connection
  - Larger output current: parallel connection

- To improve the AC side current waveform and DC side voltage waveform
Phase-shift connection of multiple rectifiers

Parallel connection

12-pulse rectifier realized by paralleled 3-phase bridge rectifiers
Phase-shift connection of multiple rectifiers

Series connection

12-pulse rectifier realized by
series 3-phase bridge rectifiers
Quantitative analysis of 12-pulse rectifier

Voltage
- Average output voltage
  - Parallel connection: \( U_d = 2.34U_2 \cos \alpha \)
  - Series connection: \( U_d = 4.68U_2 \cos \alpha \)
- Output voltage harmonics
  - Only 12\( m \) harmonics exist

Input (AC side) current harmonics
- Only 12\( k \pm 1 \) harmonics exist

Connection of more 3-phase bridge rectifiers
- Three: 18-pulse rectifier (20º phase difference)
- Four: 24-pulse rectifier (15º phase difference)
Sequential control of multiple series-connected rectifiers

Circuit and waveforms of series-connected three single-phase bridge rectifiers
2.7 Inverter mode operation of rectifiers

Review of DC generator-motor system

\[
I_d = \frac{E_G - E_M}{R_\Sigma}
\]

\[
I_d = \frac{E_M - E_G}{R_\Sigma}
\]

should be avoided
Inverter mode operation of rectifiers

Rectifier and inverter mode operation of single-phase full-wave converter

\[ I_d = \frac{U_d - E_G}{R \Sigma} \]

\[ I_d = \frac{|E_M| - |U_d|}{R \Sigma} \]
Necessary conditions for the inverter mode operation of controlled rectifiers

- There must be DC EMF in the load and the direction of the DC EMF must be enabling current flow in thyristors. (In other word $E_M$ must be negative if taking the ordinary output voltage direction as positive.)

- $\alpha > 90^\circ$ so that the output voltage $U_d$ is also negative.

- $|E_M| > |U_d|$
Inverter mode operation of 3-phase bridge rectifier

Inversion angle (extinction angle) $\beta$

$\alpha + \beta = 180^\circ$
Inversion failure and minimum inversion angle

 mogelijk reasons of inversion failures

- Malfunction of triggering circuit
- Failure in thyristors
- Sudden dropout of AC source voltage
- Insufficient margin for commutation of thyristors

Minimum inversion angle (extinction angle)

\[ \beta_{\text{min}} = \delta + \gamma + \theta' \quad (2-109) \]
2.8 Thyristor-DC motor system

2.8.1 Rectifier mode of operation

2.8.2 Inverter mode of operation

2.8.3 Reversible DC motor drive system
   (four-quadrant operation)
2.8.1 Rectifier mode of operation

Waveforms and equations

\[ U_d = E_M + R \sum I_d + \Delta U \]  
(2-112)

where

\[ R \sum = R_B + R_M + \frac{3 X_B}{2\pi} \]

(for 3-phase half-wave)

Waveforms of 3-phase half-wave rectifier with DC motor load
Speed-torque (mechanic) characteristic when load current is continuous

\[ E_M = C_e n \quad (2-113) \]

For 3-phase half-wave

\[ U_d = 1.17U_2 \cos \alpha \]

\[ E_M = 1.17U_2 \cos \alpha - R \sum I_d - \Delta U \quad (2-114) \]

\[ n = \frac{1.17U_2 \cos \alpha}{C_e} - \frac{R \sum I_d + \Delta U}{C_e} \quad (2-115) \]

For 3-phase bridge

\[ n = \frac{2.34U_2 \cos \alpha}{C_e} - \frac{R \sum I_d}{C_e} \quad (2-116) \]
Speed-torque (mechanic) characteristic when load current is discontinuous

EMF at no load (taking 3-phase half-wave as example)

For $\alpha \leq 60^\circ$

$$E_0 = \sqrt{2U_2}$$

For $\alpha > 60^\circ$

$$E_0 = \sqrt{2U_2} \cos(\alpha - 60^\circ)$$

For 3-phase half-wave
Speed-torque (mechanic) characteristic when load current is discontinuous

For different $\alpha$

- The point of EMF at no load is raised up.
- The droop rate becomes steer. (softer than the continuous mode)

For 3-phase half-wave

$(\alpha_1 < \alpha_2 < \alpha_3 \leq 60^\circ, \alpha_5 > \alpha_4 > 60^\circ)$
2.8.2 Inverter mode of operation

Equations

- are just the same as in the rectifier mode of operation except that $U_d$, $E_M$, and $n$ become negative. E.g., in 3-phase half-wave

$$E_M = 1.17U_2 \cos \alpha - R \sum I_d - \Delta U \quad (2-114)$$

$$n = \frac{1.17U_2 \cos \alpha}{C_e} - \frac{R \sum I_d + \Delta U}{C_e} \quad \text{Or in another form} \quad (2-115)$$

$$E_M = -(U_{d0} \cos \beta + I_d R \sum I) \quad (2-122)$$

$$n = -\frac{1}{C_e} U_{d0} \cos \beta + I_d R \sum \quad (2-123)$$

Speed-torque characteristic of a DC motor fed by a thyristor rectifier circuit
2.8.3 Reversible DC motor drive system (4-quadrant operation)

Back-to-back connection of two 3-phase bridge circuits
4-quadrant speed-torque characteristic of Reversible DC motor drive system

\[ \alpha = \beta = \frac{\pi}{2} \]

\[ \alpha' = \beta' = \frac{\pi}{2} \]

\[ \alpha_1 = \beta_1; \quad \alpha'_1 = \beta'_1 \]
\[ \alpha_2 = \beta_2; \quad \alpha'_2 = \beta'_2 \]
Power Electronics

Supplement:
Gate Triggering Control Circuit for Thyristor Rectifiers
2.9 Gate triggering control circuit for thyristor rectifiers

Object
- How to timely generate triggering pulses with adjustable phase delay angle

Constitution
- Synchronous circuit
- Saw-tooth ramp generating and phase shifting
- Pulse generating

Integrated gate triggering control circuits are very widely used in practice.
A typical gate triggering control circuit
Waveforms of the typical gate triggering control circuit
How to get synchronous voltage for the gate triggering control circuit of each thyristor

For the typical circuit on page 20, the synchronous voltage of the gate triggering control circuit for each thyristor should be lagging 180° to the corresponding phase voltage of that thyristor.